(8.83) Equilibrium conditions: equality of gibbs free energies

(a) $2\mu_H = \mu_{H_2}$

(b) $2\mu_{CO} + \mu_{O_2} = 2\mu_{CO_2}$

(c) $\mu_{CH_4} + 2\mu_{O_2} = 2\mu_{H_2O} + \mu_{CO_2}$

(d) $\mu_{H_2SO_4} = 2\mu_{H^+} + \mu_{SO_4}$

(e) $2\mu_p + 2\mu_n = \mu_{He}$

(8.84) eq 8.108

$$\frac{P_{NH_3}^2 (P^\circ)^2}{P_{N_2} P_{H_2}^3} = K \equiv 6.9 \cdot 10^{-5}$$

$$\frac{\left(\frac{P_{NH_3}}{P^\circ}\right)^2}{\left(\frac{P_{N_2}}{P_0}\right)\left(\frac{P_{H_2}}{P_0}\right)^3} = K$$

?

(8.85) $K \equiv e^{-\Delta G / RT}$

$$\ln K = -\frac{\Delta G}{RT}$$

$$\frac{1}{dT}(\ln K) = +\frac{\Delta G}{RT^2} - \frac{1}{RT}\frac{d\Delta G}{dT}$$

$dU = -pdV + SdT$

$G = U + PV - ST$

$\Delta G = -pdV + SdT + pdV + Vdp - SdT - TdS$

$= Vdp - TdS$

$$\frac{d}{dT}(\ln k) = \frac{1}{RT^2}\left(\Delta G - T\frac{d\Delta G}{dT}\right)$$

$$= \text{[illegible]}$$

Is $\frac{d\Delta G}{dT} = -S$ yes.

Then $\Delta G - T\frac{d\Delta G}{dT} = \Delta G + ST = \Delta H$

$$\therefore \quad \frac{d}{dT}(\ln k) = \frac{\Delta H}{RT^2}$$

5.86 $N_2 + 3H_2 \leftrightarrow 2NH_3$ $\quad T_1 = 373\ k$ $\quad T_2 = 273 + 500 = 773\ k$

$$\Delta H^\circ = 2\Delta_f H(NH_3) - 3\Delta_f H^\circ(H_2) - \Delta_f H^\circ(N_2)$$

$$= 2(-46.11) \leftarrow -90\ kJ$$

eq 5.69 H_2O

$$\mu_A = \mu_0(T,P) - \frac{N_B}{N_A} kT$$

eq 5.72 for $H^+ + OH^-$

$$\mu_B = \mu^\circ(T,P) + kT \ln m_B \qquad m = \frac{\text{moles solute}}{\text{kilogram solvent}}$$

$$\rightarrow \mu^\circ_{H_2O}(T,P) = \mu^\circ_{H^+} + kT \ln m_{H^+} + \mu^\circ_{OH^-} + kT \ln m_{OH^-}$$

$$\rightarrow -N_A(\mu^\circ_{H^+} + \mu^\circ_{OH^-} - \mu^\circ_{H_2O}) = N_A kT \ln(m_{H^+} m_{OH^-})$$

$$-\Delta G^\circ = RT \ln(m_{H^+} m_{OH^-})$$

$$pH \equiv -\log_{10} m_{H^+}$$

(5.87)

$$H_2SO_4 \rightarrow H^+ + HSO_4^- \qquad k = 10^2$$

$$HSO_4^- \longleftrightarrow H^+ + SO_4^{2-} \qquad k = 10^{-1.9}$$

(a) Based on eq 5.114

$$m_{H^+} + m_{HSO_4^-} = 10^2 \Rightarrow m_{H^+} = 10 = m_{HSO_4^-}$$

$\equiv \frac{\text{moles } \{H^+, HSO_4^-\}}{\text{kilograms solvent.}}$ quite a ~~lot~~ few moles !!

I would assume at a pH equivalent to this

$$pH \equiv -\log_{10} m_{H^+} \cong -\log_{10}(10) = -1.$$

(b) If $m_{HSO_4^-} = 5 \cdot 10^{-5}$ mol/kg

Then ?

(c) k for that reaction is $10^{-14} \Rightarrow m_{H^+} = 10^{-7} \ll m_{H^+}$ due to Sulfuric acid.

(d) ?

(5.88) $\frac{\partial(\Delta G^\circ)}{\partial P}$

$$\left(\frac{\partial(\Delta G)}{\partial P}\right)_T = V$$

$G = U + PV - ST$

$\Delta G = \Delta U + \Delta P \cdot V + V\Delta P - \Delta S \cdot T - S\Delta T$

$= -P\Delta V + S\Delta T + \Delta P V + V\Delta P - \Delta S \cdot T - S\Delta T$

$= V\Delta P - \Delta S \cdot T$

Show that for volumes of liquids in solution this # V is small.

$$\mu_{ges} = \mu_{Solute}$$

$$\mu^{\circ}_{ges}(T) + kT \ln(p/p^{\circ}) = \mu^{\circ}_{Solute}(T,p) + kT \ln m_B$$

$$N_A(\mu^{\circ}_{Solute} - \mu^{\circ}_{ges}) = N_A kT \left(\ln(p/p^{\circ}) - \ln(m_B) \right)$$

$$\underbrace{\qquad\qquad}_{\Delta G^{\circ}} = RT \ln\left(\frac{p/p^{\circ}}{m}\right)$$

$$\frac{m}{p/p^{\circ}} = e^{-\Delta G^{\circ}/RT}$$

Prob 5.89 — M 217 Schneider — 01-05-03 1

$\Delta H^\circ = -11.7\,kJ$

Van't Hoff equation:

$$\frac{d(\ln k)}{dT} = \frac{\Delta H^\circ}{RT^2}$$

$\Rightarrow$ if ΔH° is independent of temperature

$$\ln k(T_2) - \ln k(T_1) = \frac{\Delta H^\circ}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\ln(k(372\,k)) - \ln(1/180) = \frac{(-11.7\cdot 10^3\,J)}{(8.314\,J/k)}\left(\frac{1}{273} - \frac{1}{373}\right)$$

$k(372\,k) = \ldots$

How calculate $k(272\,k)$? Can I not just use 5.124?

5.90 $\Delta G^\circ = -1307.67 - (-856.64) - 2(-237.13)$ kJ

(a) $T = 273 + 25$ k

$$\exp\left(-\frac{\Delta G^\circ}{RT}\right) = \ldots$$

Then $\frac{m}{P/p^\circ} = \ldots$ what is the corresponding P expression?

think $P/p^\circ \cong 1$, but why?

(b) compute $k(372\,k)$ By ~~last~~ van't Hoff eq.

+ ΔH°

5.91 Using eq 5.123 calculate m, i.e.:

$$\frac{m}{(P/P^\circ)} = e^{-\Delta G^\circ/RT}$$

$P = 3.4 \cdot 10^{-4}$ bar

$m = \ldots$ molality of carbonic acid

Given that "neutral" H_2O has this much.

$$\Delta G^\circ = -623.08 - 1(-237.13) - 1(-394.36) \quad [\text{kJ}]$$

$$= 8.41 \text{ kJ}$$

$$k = e^{-\Delta G^\circ/RT} = .03167$$

$$\Rightarrow m = 1.07 \cdot 10^{-5} \frac{\text{moles}}{\text{kg solvent}}$$

Then the entirety aqueous transformation is governed by:

$$m_{H^+} \cdot m_{HCO_3^-} = e^{-\Delta G^\circ/RT}$$

w/

$$\mu_{H_2CO_3} = \mu_{H^+} + \mu_{HCO_3^-}$$

$$\mu^\circ_{H_2CO_3} + kT \ln(m_{H_2CO_3}) = \mu^\circ_{H^+} + kT \ln(m_{H^+}) + \mu^\circ_{HCO_3^-} + kT \ln(m_{HCO_3^-})$$

$$N_A\left(\mu^\circ_{H_2CO_3} - \mu^\circ_{H^+} - \mu^\circ_{HCO_3^-}\right) = \text{[illegible]}$$

$$= N_A k T \ln\left[\frac{m_{H^+} m_{HO_3^-}}{m_{H_2CO_3}}\right]$$

$$\Rightarrow \exp\left(-\frac{\Delta G^\circ}{RT}\right) = \frac{m_{H_2CO_3}}{m_{H^+} m_{HO_3^-}}$$

Assuming that $m_{H^+} = m_{HO_3^-}$ . . .

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$$\mu_H = \mu_p + \mu_e$$

$$\mu_H^\circ + kT\ln\left(\frac{P_H}{P_0}\right) = \mu_p^\circ + kT\ln\left(\frac{P_p}{P_0}\right) + \mu_e^\circ + kT\ln\left(\frac{P_e}{P_0}\right)$$

$$\Rightarrow \mu_p^\circ + \mu_e^\circ - \mu_H^\circ = kT\ln\left[\frac{P_H/P_0}{\left(\frac{P_p}{P_0}\right)\left(\frac{P_e}{P_0}\right)}\right]$$

$$= kT\ln\left[\frac{P_0 P_H}{P_p P_e}\right] \quad \text{eq } 5.126 \checkmark$$

$$\mu = -kT\ln\left[\frac{V}{N}\left(\frac{2\pi m kT}{h^2}\right)^{3/2}\right]$$

$$PV = NkT$$

$$\frac{V}{N} = \frac{kT}{P}$$

5.92

(a) $\frac{N_p}{N_H^\circ}$

$$N_H^\circ = N_H(T) + N_p(T)$$

$$PV = kNT \qquad P = \frac{kNT}{V}$$

$$N = \frac{PV}{kT}$$

P_{H° ?

(b) ?

(c) ?

(d)

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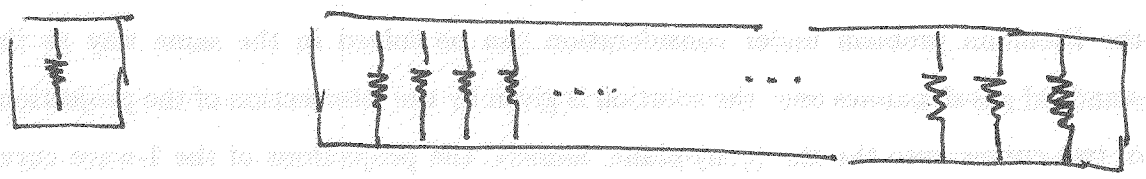
(6.1)



$$\Omega(N,q) = \binom{q+N-1}{q}$$ Multiplicity of an einstein solid w/ N oscillators & q units of energy

$$\Omega_{AB} = \Omega\binom{q+N_A+N_B-1}{q}$$

$N_A = 1$; $N_B = 100$; $q = 500$; ? I should be able to do this problem...

(6.2) $$P(s) = \frac{1}{Z} e^{-E(s)/kT}$$

$P(E) = ?$

since there are $\Omega(E)$ multiplicities of states w/ energy E one gets

$$P(E) = \frac{\Omega(E)}{Z} e^{-E/kT}$$ ~~$S(N(E)) = k \ln$~~

$S = k \ln \Omega(E)$

w/ E = energy of system in state s. $\Omega = \exp\{S/k\}$

so $$P(E) = \frac{1}{Z} e^{\frac{S}{k}} e^{-\frac{E}{kT}} = \frac{1}{Z} \exp\left\{-\frac{(E-TS)}{kT}\right\}$$

$$P(E) = \frac{1}{Z} \exp\left\{ -\frac{E}{kT} \right\}$$

$$P(E) = \frac{1}{Z} e^{-E/kT}$$

(6.3) 2 ground states

$E_0 = 0$ eV

$E_1 = 2$ eV

$$Z = \sum_s e^{-E(s)/kT}$$

$$= e^{-0} + e^{-2eV/kT} = 1 + e^{-\frac{2}{kT}}$$

$[kT] = eV$

(6.4) $$Z \cong \sum e^{-E(s)/kT} \qquad E(s) =$$

$$= \oint \sum_s P(s) = \int_0^\infty P(s)\,ds = \int_0^\infty e^{-s/kT}\,ds = \cdots$$

(6.5) (a)

$$Z = \sum_s e^{-\frac{E(s)}{kT}}$$

$$= e^{-\frac{.05}{kT}} + 1 + e^{\frac{.05}{kT}} = \cdots$$

$T = 300K$

(b)

$$P(s_1; E_1 = -.05\,eV) = \frac{e^{-.05/kT}}{Z} \qquad P(s_3; E_3 = +.05\,eV) = \frac{e^{+\frac{.05}{kT}}}{Z}$$

$$P(s_2; E_2 = 0) = \frac{1}{Z}$$

(c) Z will change since E has changed

The prob. will not change.

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Prob 6.6

$$\frac{P(s_2)}{P(s_1)} = \frac{e^{-E(s_2)/kT}}{e^{-E(s_1)/kT}} = \exp\left\{-\frac{(E(s_2)-E(s_1))}{kT}\right\}$$

$$= \exp\left\{\frac{-(10.2\,eV)}{(8.62\cdot10^{-5}\,eV/K)(293\,K)}\right\} = \ldots$$

Then since 1st excited state has 4 levels all w/ the same energy we get

4 x (this #)

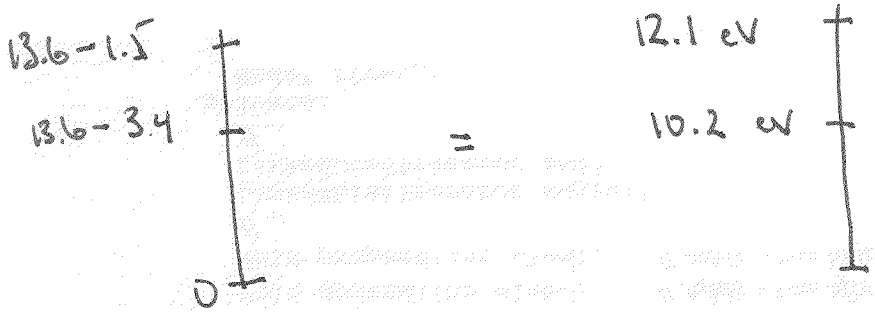
Scm w/ $T = 9500\,K$.

6.7 Think Both $p(s_2)$ & $p(s_1)$ would get multiplied by a constant & Z

6.8 The methods of this section apply to a given system embedded in a "universe" at ~~another~~ a temperature T.

Don't know?

6.9 $$Z \equiv \sum_s e^{-E(s)/kT} =$$



(a)

$$Z = 1 + e^{\frac{-10.2}{kT}} + e^{\frac{-12.1}{kT}}$$

↑ is there a multiplicity coefficient here?

$T = 5800\,k$

From Appendix A: $E(n) = -\frac{13.6\ eV}{n^2} + 13.6\ eV$

(b)

How do I know

$$= 13.6\ ev\left[1 - \frac{1}{n^2}\right]$$

$$\sum_n \exp\left\{\frac{13.6}{kT}\left(1 - \frac{1}{n^2}\right)\right\}$$ diverges?

$$= \exp\left\{\frac{13.6}{kT}\right\} \sum_n \exp\left\{-\frac{13.6}{kTn^2}\right\}$$

$\lim_{n\to\infty} \exp\left\{\frac{-13.6}{kTn^2}\right\} \neq 0$

$\therefore$ series diverges

(c) $R_n = a_0 n^2$

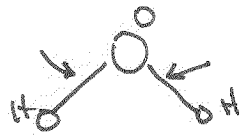
dV for large n's then becomes

$$dV = a_0((n+1)^2 - n^2) = a_0(n^2 + 2n + 1 - n^2) = a_0(2n+1) = \Theta(n)$$

$\therefore$ $PdV = \Theta(n)$ for large n

$\therefore$ P-V work ~~associated~~ by that the atoms does when an atom transfers levels is not negligible for high energy level transfers. Don't see ~~why~~ where this P-V work affects $E(s)$?

(6.10)



$\nu_0 = 4.8 \cdot 10^{13}$ Hz $\qquad E(s) = \left(\frac{2s+1}{2}\right) h\nu$

(a) $Z = \sum_s e^{-\frac{E(s)}{kT}} = \sum_{s=0}^{\infty} \exp\left\{\left(\frac{2s+1}{2} \cdot h\nu_0\right)\right\} = \dots$

$$P(s_1) = \frac{\exp\left\{\frac{h\nu_0}{kT}\right\}}{Z}$$

Think in the perturbation formulation if we had degeneracy at a given energy level then we must add in that # of $\exp\{-\frac{E(s)}{kT}\}$ factors.

(b) ...

(6.11) $E = m\mu B \qquad \mu = 1.03 \cdot 10^{-7} \frac{eV}{T} \qquad B = 0.63\,T$

$$Z = \sum_s e^{-E(s)/kT} = e^{+\frac{3}{2}\mu B/kT} + e^{\frac{1}{2}\mu B/kT} + e^{-\frac{1}{2}\mu B/kT} + e^{-\frac{3}{2}\mu B/kT}$$

$$P\left(-\tfrac{3}{2}\right) = \frac{e^{+\frac{3}{2}\frac{\mu B}{kT}}}{Z}, \dots$$

reversing $B \leftrightarrow -B \Leftrightarrow -T$

(6.12) $\frac{P(S^*)}{P(S_0)} = \frac{3}{10}$ By experimental observation.

$= \frac{3 e^{-\frac{E(S^*)}{kT}}}{e^{-\frac{E(S_0)}{kT}}} = .3$ By Boltzmann statistics. & knowledge of degeneracy of 1st excited state

$\rightarrow \exp\left\{-(E(S^*)-E(S_0))/kT\right\} = .1$

$\Rightarrow \exp\left\{\frac{-4.7\cdot 10^{-4}\,eV}{kT}\right\} = 1 \rightarrow t = \ldots$

(6.13) $E_n - E_p = (2.3\cdot 10^{-30}\,kg)\,c^2$

$T = 10^{11}\,K.$

$\frac{P(n)}{P(p)} = \exp\left\{-\frac{(E_n - E_p)}{kT}\right\} = \ldots$ = fraction of nucleons that were neutrons.

W no degeneracy.

(6.14)

--o-- molecule of air at z

--o-- molecule of air at z = 0 (sea level)

$\frac{N(z)}{N(0)} = \frac{P(z)}{P(0)} = \exp\left\{-\frac{(E(z)-E(0))}{kT}\right\}$

01-06-03

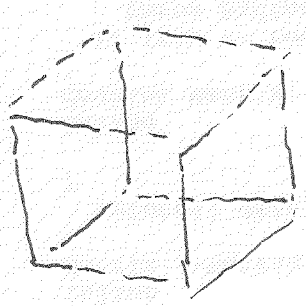
Then w/ $E(z) = mgz$ potential energy

$$N(z) = N_0 \exp\left\{-\frac{mgz}{kT}\right\}$$ to relate to pressure

$$PV = kNT$$

$$\Rightarrow N = \frac{PV}{kT}$$

What does V represent? A fixed volume that one moves up to height z



then

$$\frac{P(z)V}{kT} = \frac{P(0)V}{kT} \exp\left\{-\frac{mgz}{kT}\right\}$$

Same as eq 1.16 —

6.15

(a) $\bar{E} = \dfrac{4.0eV + 3.1eV + 2.4eV + 1.6eV}{10}$

(b) $P(0eV) = \frac{4}{10}$ $P(4eV) = \frac{2}{10}$

$P(1eV) = \frac{3}{10}$ $P(6eV) = \frac{1}{10}$

(c) $\bar{E} = (\frac{4}{10}).0eV + (\frac{3}{10}).1eV + (\frac{2}{10}).4eV + (\frac{1}{10}).6eV$

6.16 $Z = \sum_s e^{-\frac{E(s)}{kT}} = \sum_s e^{-E(s)\beta}$

$\frac{\partial Z}{\partial \beta} = \sum_s -E(s) e^{-E(s)\beta}$

Since $\bar{E} = \frac{1}{Z}\sum_s E(s) e^{-E(s)\beta}$

we see that $\bar{E} = \frac{-1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{\partial \ln Z}{\partial \beta}$

6.17 (a) $E_i - \bar{E} = \begin{cases} 7-3 = 4 \\ 4-3 = 1 \\ 4-3 = 1 \\ 0-3 = -3 \\ 0-3 = -3 \end{cases}$ ΔE_i ; $\Delta E_i^2 = \begin{cases} 16 \\ 1 \\ 1 \\ 9 \\ 9 \end{cases}$

$$\overline{(\Delta E_i)^2} = \frac{1}{5}(16+1+1+9+9) = \frac{36}{5} = 7.2 \equiv \sigma_E^2$$

$$\sigma_E = \sqrt{7.2}$$

$$\sigma^2 = \text{Avg}\,(E_i - \bar{E})^2$$

$$= \text{Avg}\,(E_i^2 - 2E_i\bar{E} + \bar{E}^2)$$

$$= \overline{E^2} - 2\bar{E}^2 + \bar{E}^2 = \overline{E^2} - \bar{E}^2$$

(d) $\sigma^2 =$ ...

Prob 6.18

$$Z = \sum_s \exp(-E(s)\beta) \qquad \overline{E^2} = \sum_s E^2(s)\frac{e^{-E(s)\beta}}{Z}$$

$$= \frac{1}{Z}\sum_s E(s)^2 e^{-E(s)\beta}$$

$$\frac{\partial Z}{\partial \beta} = \sum_s -E(s)\,e^{-E(s)\beta}$$

$$\frac{\partial^2 Z}{\partial \beta^2} = \sum_s E^2(s)\,e^{-E(s)\beta}$$

$$\therefore \quad \overline{E^2} = \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2}$$

Since $\sigma^2 = \overline{E^2} - \bar{E}^2 = \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2} - \left(-\frac{1}{Z}\frac{\partial Z}{\partial \beta}\right)^2$

Since $\bar{E} = \frac{1}{Z}\frac{\partial Z}{\partial \beta}$

$\sigma^2 = \frac{1}{Z}\frac{\partial}{\partial \beta}(\bar{E}) - \bar{E}^2$

Something is fishy... try again

$\sigma_E^2 = \overline{E^2} - (\bar{E})^2 = \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2} - \left(-\frac{1}{Z}\frac{\partial Z}{\partial \beta}\right)^2$

$= \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2} - \frac{1}{Z^2}\left(\frac{\partial Z}{\partial \beta}\right)^2$

$= \frac{\partial}{\partial \beta}\left(\frac{1}{Z}\frac{\partial Z}{\partial \beta}\right) = \frac{\partial}{\partial \beta}\left(-\bar{E}\right)$

w/ $\beta = \frac{1}{kT}$

$\frac{\partial}{\partial \beta} = \frac{\partial T}{\partial \beta}\frac{\partial}{\partial T}$ $\qquad$ $\frac{\partial \beta}{\partial T} = \frac{-1}{kT^2}$

$\therefore \quad \sigma_E^2 = \left(\frac{1}{kT^2}\right)^{-1}\frac{\partial \bar{E}}{\partial T} = \left(\frac{1}{kT^2}\right)^{-1} C = kT^2 C$

$\sigma_E = T\sqrt{kC} = kT\sqrt{C/k}$ ✓

(6.19) $\frac{\sigma_E}{\bar{E}}$ = fractional fluctuation in Energy

Prob 6.19

Einstein solid high temp limit $\Omega(N,q) \cong \left(\frac{eq}{N}\right)^N$

$$S = Nk\left[\ln(q/N) + 1\right]$$

$$U = NkT$$

$$C_V = Nk$$

Based on prob 6.18

$$\sigma_E = kT\sqrt{\frac{C}{k}} = kT\sqrt{N} =$$

standard deviation of the energy of systems

$$\text{Fractional Fluctuation in energy} = \frac{\sigma_E}{U} = \frac{kT\sqrt{N}}{NkT} = \frac{1}{\sqrt{N}}$$

$f(N=1) = 1$

$f(N=10^4) = 10^{-2}$

$f(N=10^{20}) = 10^{-10}$.

6.20

$$1-x \;\overline{)\; 1+0x+0x^2+0x^3+0x^4+\cdots}\;\; 1+x+x^2+\cdots$$

$$1-x$$

$$x+0x^2+0x^3+\cdots$$

$$x-x$$

$$+x^2+0x^3+0x^4+\cdots$$

$$+x^2-x^3$$

$$x^3$$

(a) $|x|<1$

(b) $$Z=\sum_s e^{-E(s)\beta}$$

$$E(s)=shf \qquad s=0,1,2\cdots$$

$$=\sum_{s=0}^{\infty} e^{-shf\beta}=\sum_{s=0}^{\infty}(e^{-hf\beta})^s=\frac{1}{1-e^{-hf\beta}}\cdot\frac{e^{+hf\beta/2}}{e^{+hf\beta/2}}$$

$$=\frac{e^{hf\beta/2}}{2\sinh(hf\beta)}$$

(c) $$\bar{E}=-\frac{1}{Z}\frac{\partial Z}{\partial\beta}=-\frac{1}{Z}\frac{\partial}{\partial\beta}\left(\frac{1}{1-e^{+hf\beta}}\right)$$

$$=\frac{-1}{Z}\frac{1}{(1-e^{-hf\beta})^2}(+hf)=\frac{-1}{Z}Z^2hf$$

$$=-Zhf=\frac{-hf}{1-e^{+hf\beta}}$$

(d) $U = N\bar{E} = \dfrac{-h\nu N}{1-e^{-h\nu\beta}}$

lab [illegible] 3.25

(e) $C = \dfrac{\partial \bar{E}}{\partial T} = \dfrac{\partial}{\partial T}\left(\dfrac{-h\nu N}{1-e^{-h\nu\beta}}\right) = \cdots$

6.21 $E_n \approx \epsilon(1.03n - 0.03n^2) \qquad n = 0, 1, 2 \cdots$

$$Z = \sum_s e^{-E(s)/kT} = \sum_{n=0}^{\infty} e^{\frac{-\epsilon(1.03n-0.03n^2)}{kT}}$$ partition fn

$$\bar{E} = \frac{-1}{Z}\frac{\partial Z}{\partial \beta} + \frac{\partial Z}{\partial \beta} = \sum_{n=0}^{\infty} e^{-\epsilon(1.03n-0.03n^2)\beta}\left(-\epsilon(1.03n-0.03n^2)\right)$$

$$= -\epsilon \sum_{n=0}^{\infty} (1.03n - 0.03n^2)\, e^{-\epsilon(1.03n-0.03n^2)/\beta}$$

$C = \dfrac{\partial \bar{E}}{\partial T} = \cdots$

do summation only up to $n = 15$.

(6.22)

$$\mu_z = -j\delta\mu,\ (-j+1)\delta\mu,\ (-j+2)\delta\mu,\ \dots,\ (j-1)\delta\mu,\ j\delta\mu$$

$\delta\mu$ constant $= \Delta\mu_z$

(a)

$$1-x \;\overline{)\; 1 + 0x + 0x^2 + 0x^3 + \cdots + 0x^{n-1} + 0x^n + 0x^{n+1}}$$

$$\underline{1 - x}$$

$$x$$

$$\underline{x - x^2}$$

$$x^2 + 0x^3 + \cdots + 0x^n + x^{n+1}$$

$$\vdots \qquad x^3 + 0x^4 + \dots$$

$$\underline{\qquad}$$

$$x^n + x^{n+1}$$

$$\underline{x^n - x^{n+1}}$$

$$2x^{n+1} \quad ?$$

$$\{-5, 4\}$$

$$-2, 1$$

$$\underbrace{-2, -1, 0, 1}_{4}$$

↯ do w/ ∞ series ...

(b)

$$Z = \sum e^{-E_s\beta} = \sum_{s=-j}^{j-1} e^{-s\delta\mu\beta} = \sum_{s=0}^{2j-1} e^{-\delta\mu\beta(s-j)}$$

$$= e^{+\delta\mu\beta j} \sum_{s=0}^{2j-1} e^{-\delta\mu\beta s} = e^{\delta\mu\beta j}\,\frac{1 - e^{-\delta\mu\beta(2j)}}{1 - e^{-\delta\mu\beta}}$$

$$= \frac{e^{\delta\beta j} - e^{-\delta\beta j}}{1 - e^{-\delta\beta}} \cdot \frac{e^{\delta\beta/2}}{e^{\delta\beta/2}}$$

$$= e^{\Delta\beta(j+1/2)} - e^{-\Delta\beta(j+1/2)}$$

6.22

$\mu_z = -j\delta_\mu, (-j+1)\delta_\mu, \ldots, (j-1)\delta_\mu, j\delta_\mu$

(a) ✓

(b) $Z = \sum_S e^{-E(S)\beta}$

$E(S) = -\mu_z(S)B = -S\delta_\mu B$

$\therefore \quad Z = \sum_{S=-j}^{+j} \exp\{-S\delta_\mu B\beta\}$ let $b = \beta\delta_\mu B$

$$= \sum_{S=-j}^{+j} \exp\{-Sb\} = \sum_{S=0}^{2j} \exp\{-(S-j)b\}$$

$$= \sum_{S=0}^{2j} e^{-Sb} \cdot e^{jb} = e^{jb} \sum_{S=0}^{2j} e^{-Sb}$$

$$= e^{jb}\left(\frac{1 - e^{-b(2j+1)}}{1 - e^{-b}}\right) = \frac{e^{jb} - e^{-b(j+1)}}{1 - e^{-b}} \cdot \frac{e^{b/2}}{e^{b/2}}$$

$$= \frac{e^{b(j+1/2)} - e^{-b(j+1/2)}}{e^{b/2} - e^{-b/2}} = \frac{\sinh(b(j+1/2))}{\sinh(b/2)}$$

(c)

$$\bar{\mu}_z = \sum_s \mu_z(s) P(s) = \frac{1}{Z} \sum_s \mu_z(s) e^{-E(s)\beta}$$

$$\mu_z(s) = s\delta_M \qquad S = -j, -j+1, -j+2, \cdots, j-1, j$$

$$E(s) = -\mu_z(s)B = -s\delta_M B$$

$$\bar{\mu}_z = \frac{1}{Z} \sum_{s=-j}^{+j} (s\delta_M) e^{+s\delta_M B\beta} = \frac{\delta_M}{Z} \sum_{s=-j}^{+j} s e^{sb} \qquad b = \delta_M B\beta$$

$$= \frac{\delta_M}{Z} \sum_{s=-j}^{+j} \frac{\partial}{\partial b} e^{sb} = \frac{\delta_M}{Z} \frac{\partial}{\partial b} \sum_{s=-j}^{+j} e^{sb}$$

$$= \frac{\delta_M}{Z} \frac{\partial}{\partial b} \left[\sum_{s=0}^{2j} e^{(s-j)b} \right] = \frac{\delta_M}{Z} \frac{\partial}{\partial b} \left[e^{-jb} \sum_{s=0}^{2j} e^{sb} \right]$$

$$= \frac{\delta_M}{Z} \frac{\partial}{\partial b} \left[e^{-jb} \left(\frac{1 - e^{b(2j+1)}}{1 - e^{b}} \right) \right]$$

$$= \frac{\delta\mu}{Z}\frac{\partial}{\partial b}\left[\frac{e^{-jb} - e^{b(j+1)}}{1-e^{b}}\right]$$

$$= \frac{\delta\mu}{Z}\frac{\partial}{\partial b}\left[\frac{e^{-b(j+1/2)} - e^{b(j+1/2)}}{e^{-b/2} - e^{b/2}}\right]$$

$$= \frac{\delta\mu}{Z}\frac{\partial}{\partial b}\left[\frac{\sinh(b(j+1/2))}{\sinh(b/2)}\right]$$

$$= \frac{\delta\mu}{Z}\cdot\left[(j+1/2)\frac{\cosh(b(j+1/2))}{\sinh(b/2)} - \frac{\sinh(b(j+1/2))\cosh(b/2)\cdot 1/2}{(\sinh(b/2))^2}\right]$$

$$= \delta\mu\left[(j+1/2)\frac{\sinh(b/2)}{\sinh(b(j+1/2))}\frac{\cosh(b(j+1/2))}{\sinh(b/2)} - \frac{1}{2}\frac{\sinh(b/2)}{\sinh(b(j+1/2))}\frac{\sinh(b(j+1/2))\cosh(b/2)}{(\sinh(b/2))^2}\right]$$

$$= \delta\mu\left[(j+1/2)\coth(b(j+1/2)) - \frac{1}{2}\coth(b/2)\right]$$

Then $M = N\bar{\mu}_z$

$$= N\delta\mu\left[(j+1/2)\coth(b(j+1/2)) \quad \ldots\right]$$

(d) $T \to 0$ $M \to 0$ why is this expected...?

(e) $M \sim \frac{1}{T}$ does this not contradict the plots obtained?

$$\coth(x) = \frac{\cosh(x)}{\sinh(x)} \approx$$

~~$\cosh(x) = 1 + \frac{x^2}{2} + \frac{x^4}{4!}$~~

$$\cosh(x) = 1 + \frac{x^2}{2} + \frac{x^4}{4!} + \cdots$$

$$\sinh(x) = \frac{e^{x} - e^{-x}}{2} = \frac{1}{2}\sum_{k=0}^{\infty} \frac{x^k}{k!} - \frac{1}{2}\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k!}$$

$$= \sum_{k=0}^{\infty} \frac{1}{2}\left(1 - (-1)^k\right) \frac{x^k}{k!}$$

$$= \sum_{k=1,3,5,\cdots}^{\infty} \frac{x^k}{k!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

Thus $\coth(x) \approx \dfrac{1 + x^2/2 + x^4/4!}{x + x^3/3! + \cdots} = \dfrac{1}{x + x^3/3!} + \dfrac{x^2/2}{x + x^3/3!} + \cdots$

$$\approx \frac{1}{x} + \frac{x}{2} + O(x^2)$$

$$= \frac{1}{x(1+x^2/6)} + \frac{x}{2}\frac{1}{(1+x^2/6)} + \mathcal{O}(x^3)$$

$$= \frac{1}{x}\left[\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k}}{6^k}\right] + \frac{x}{2}\left[\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k}}{6^k}\right] + \mathcal{O}(x^3)$$

$$= \frac{1}{x}\left[1 - \frac{x^2}{6} + \mathcal{O}(x^4)\right] + \frac{x}{2}\left[1 - \mathcal{O}(x^2)\right]$$

$$= \frac{1}{x} - \frac{x}{6} + \frac{x}{2} + \mathcal{O}(x^2)$$

$$= \frac{1}{x} + x\left[\frac{1}{2} - \frac{1}{6}\right] + \mathcal{O}(x^2)$$

$$= \frac{1}{x} + x\left[\frac{2}{6}\right] + \mathcal{O}(x^2)$$

$$= \frac{1}{x} + \frac{x}{3} + \mathcal{O}(x^2)$$

Then $T \to \infty \Rightarrow \beta \to 0 = b \to 0$

$$\Pi \sim N\delta_m\left[(j+1/2)\circ\left[\frac{1}{b(j+1/2)} + \frac{b(j+1/2)}{3}\right] - \frac{1}{2}\left[\frac{1}{b/2} + \frac{b/2}{3}\right]\right]$$

$$M \sim N\delta_M \left[\frac{1}{6} + \frac{b}{3}(j+1/2)^2 - \frac{1}{6} - \frac{b}{12} \right]$$

$$= \frac{N\delta_M}{3}\left[b(j+1/2)^2 - \frac{b}{4} \right]$$

$$= \frac{N\delta_M b}{3}\left[(j+1/2)^2 - (1/2)^2 \right] \sim \frac{1}{T} \quad \text{Curie law.}$$

(4) $M = N\delta_M \left[\cosh(b) - \frac{1}{2}\cosh(1/4) \right]$ ..

6.23 $\epsilon = .00024\ eV$

$$Z_{rot} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT}$$

$$Z_{rot}(300k) = \sum_{j=0}^{\infty} (2j+1) e^{-\frac{j(j+1)(2.4\cdot10^{-4})ev}{(8617\cdot10^{-5}\,ev/k)(300k)}}$$

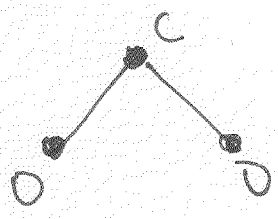
$=$

$$Z_{rot} = \frac{kT}{\epsilon} \quad \text{Approx...}$$

6.24 $Z_{rot} \not\approx kT$ O_2 has 2 indistinguishable atoms

$$Z_{rot} \sim \frac{kT}{2\epsilon}$$

6.25



$Z_{rot} \approx \frac{kT}{\epsilon}$ if analysis holds ...

6.26 $kT \ll \epsilon$

$$Z_{rot} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT}$$

$$\cong 1 + 3e^{-2\epsilon/kT} = 1 + 3e^{-2\epsilon\beta}$$

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = \frac{-1}{Z}\frac{\partial}{\partial \beta}\left(1 + 3e^{-2\epsilon\beta}\right) = -\frac{1}{Z}\left(3(-2\epsilon)e^{-2\epsilon\beta}\right)$$

$$= -\frac{1}{Z}(-2\epsilon)\left(3e^{-2\epsilon\beta}\right)$$

$$= -\frac{1}{Z}(-2\epsilon)(Z-1) \cong \cdots + 2\epsilon\left(1 - \frac{1}{Z(\beta)}\right)$$

$$\bar{C}_V = \frac{\partial \bar{E}}{\partial T} = \frac{\partial \beta}{\partial T}\frac{\partial \bar{E}}{\partial \beta} \qquad \beta = \frac{1}{kT}$$

$$= -\frac{\beta}{T}\frac{\partial \bar{E}}{\partial \beta} \qquad \frac{\partial \beta}{\partial T} = \frac{-1}{kT^2} = -\frac{1}{T}\beta$$

$$\therefore \quad \bar{C}_V = -\frac{\beta}{T}\left(\frac{1}{Z^2}\frac{\partial Z}{\partial \beta}\right)$$

$$\bar{C}_V(T \to 0) \stackrel{?}{=} 0$$

$$\bar{C}_V =$$

6.26

$$Z_{rot} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT}$$

low temp limit $\frac{\epsilon}{kT} \gg 1$

$$\approx 1 + 3e^{-2\epsilon/kT} = 1 + 3e^{-2\epsilon\beta}$$

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{1}{Z}\frac{\partial}{\partial \beta}(1 + 3e^{-2\epsilon\beta}) = -\frac{1}{Z}\left(3(-2\epsilon)e^{-2\epsilon\beta}\right)$$

$$= \frac{6\epsilon e^{-2\epsilon\beta}}{1 + 3e^{-2\epsilon\beta}}$$

$$\bar{C}_{V3} = \frac{\partial \bar{E}}{\partial T} \; ; \quad \bar{C} = \frac{\partial \bar{E}}{\partial T} = \frac{\partial \beta}{\partial T}\frac{\partial \bar{E}}{\partial \beta}$$

$$\beta = \frac{1}{kT}$$

$$\frac{\partial \beta}{\partial T} = \frac{-1}{kT^2}$$

$$= -\frac{\beta}{T}\frac{\partial \bar{E}}{\partial \beta}$$

$$= -\frac{1}{T}\beta$$

$$k\left(\frac{\Delta E}{kT}\right)^2 = C.$$

so

$$\frac{\partial \bar{E}}{\partial \beta} = \frac{-12\epsilon^2 e^{-2\epsilon\beta}}{1 + 3e^{-2\epsilon\beta}} - \frac{6\epsilon e^{-2\epsilon\beta} \cdot (-6\epsilon e^{-2\epsilon\beta})}{(1 + 3e^{-2\epsilon\beta})^2}$$

$$= \frac{-12\epsilon e^{-2\epsilon\beta}}{1 + 3e^{-2\epsilon\beta}} + \frac{36\epsilon^2 e^{-4\epsilon\beta}}{(1 + 3e^{-2\epsilon\beta})^2}$$

$$= \frac{6te^{-2t\beta}}{(1+3e^{-2t\beta})^2}\left[-2(1+3e^{-2t\beta}) + 6te^{-2t\beta}\right]$$

$$= \frac{6te^{-2t\beta}}{(1+3e^{-2t\beta})^2}\left(\cancel{-12t+6t} - 2 - 6e^{-2t\beta} + 6te^{-2t\beta}\right)$$

$$= \frac{6te^{-2t\beta}}{(1+3e^{-2t\beta})^2}\left(-2 - 6(1+t)e^{-2t\beta}\right)$$

$$= -\frac{12te^{-2t\beta}}{(1+3e^{-2t\beta})^2}\left(1 + 3(1+t)e^{-2t\beta}\right)$$

to simplify keeping only the lowest T dependent term $\Rightarrow \beta \ll 1$

$$\Rightarrow \bar{E} = \frac{6te^{-2t\beta}}{1+3e^{-2t\beta}} \cong 6te^{-2t\beta}\cancel{\left(1 - 3e^{-2t\beta}\right)}$$

$$= 6t\left(\cancel{e^{-2t\beta}} - 3e^{-4t\beta}\right)$$

$$C \equiv \frac{\partial \bar{E}}{\partial T} = -\frac{\beta}{T}\frac{\partial \bar{E}}{\partial \beta} = \frac{-\beta}{T}6t\left(-2te^{-2t\beta} + 12te^{-4t\beta}\right)$$

$$\bar{E} \simeq 6te^{-2t\beta}$$

Using the full expression:

(6.27) $$Z_{rot} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT} \quad \text{v.s.}$$

$$Z_{rot} = \int_0^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT}\, dj = \frac{kT}{\epsilon}$$

...

(6.28) $$Z_{rot} \cong \sum_{j=0}^{k} (2j+1) e^{-j(j+1)\epsilon/kT}$$

$$\bar{E} = \frac{-1}{Z}\frac{\partial Z}{\partial \beta} \qquad 0 \le \frac{kT}{\epsilon} \le 3 \;\rightarrow\; \frac{1}{3} \le \frac{kT}{\epsilon} \le \infty$$

- -

(6.29) $\epsilon = 0.0057\ eV$

H — D

Assume $\frac{kT}{\epsilon} \ll 1$ $\qquad \frac{\epsilon}{kT} \gg 1$

$$\rightarrow \quad T \ll \frac{\epsilon}{k} = \frac{0.0057\ eV}{(8.617 \cdot 10^{-5}\ eV/K)}$$

6.30 $\epsilon = .0076$ eV

(a) j even

$$Z_{rot} = \sum_{j=0}^{\infty} (4j+1)\, e^{-j(j+1)\epsilon/kT}$$

ϵ known

$\frac{\epsilon/k}{T}$

...

(b) $$Z_{rot} = \sum_{j>0}^{\infty} (2(2j+1)+1)\, e^{-(2j+1)((2j+1)+1)\epsilon/kT}$$

...

(c) $\frac{1}{4}$(para hy) + $\frac{3}{4}$(ortho hy)

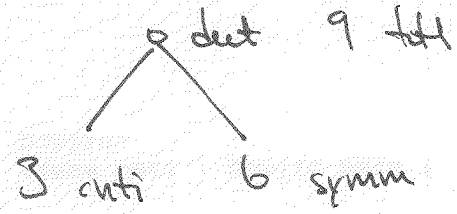
$$Z_{rot\ mix} \overset{?}{=} \frac{1}{4} Z_{rot\ para} + \frac{3}{4} Z_{rot\ ortho.}$$

(d) $$Z = \sum_{j=0}^{\infty} (4j+1)\, e^{-j(j+1)\epsilon/kT} + \sum_{j=0}^{\infty} 3\cdot(2(2j+1)+1)\, e^{-(2j+1)(2j+2)\epsilon/kT}$$

...

(e)

01-16-03 8



∴

(6.3) $E = c|q|$

Steps in Equip thm

1) Assum energy levels are discrete q_i
2) compute the partition function for a system w/ this energy

$$Z = \sum_q e^{-\beta E(q)} = \sum_q e^{-\beta c|q|} = \frac{1}{\Delta q}\sum_q e^{-\beta c|q|}\Delta q$$

$$\approx \frac{1}{\Delta q}\int_{-\infty}^{+\infty} e^{-\beta c|q|}\,dq = \frac{2}{\Delta q}\int_0^{\infty} e^{-\beta c q}\,dq = \frac{2}{\Delta q}\frac{e^{-\beta c q}}{(-\beta c)}\Bigg|_0^{\infty}$$

$$= \frac{2}{\Delta q(-\beta c)}(0-1) = \frac{2}{\Delta q \beta c}$$

Now:

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{1}{\left(\frac{2}{\Delta q \beta c}\right)}\frac{\partial}{\partial \beta}\left(\frac{2}{\Delta q \beta c}\right)$$

$$= -\frac{\Delta q \beta c}{2}\left(\frac{Z}{\Delta q c}\right)(-1\beta^{-2}) = \beta\cdot\beta^{-2} = \beta^{-1} = \underline{\underline{kT}}$$

6.32

(a) $Z = \sum_q e^{-\beta q} = \int_{-\infty}^{\infty} e^{-\beta q(x)}\, dx$

This is the definition of

$\bar{X} = \sum_i x_i P(x_i)$ or if the variable x is given continuously

w/ $P(x_i) = \dfrac{e^{-\beta U(x_i)}}{Z}$

$\bar{X} = \int_{\mathbb{R}} x P(x)\, dx = \int x \dfrac{e^{-\beta U(x)}}{Z}\, dx = \ldots$

(b) For stability $F = -\dfrac{dU}{dx}$

~~B must be 0.~~ If a linear term was not zero

$F = -\dfrac{1}{dx}\left[(x-x_0)\dfrac{dU}{dx}\Big|_{x_0} + \dfrac{(x-x_0)^2}{2}\dfrac{d^2U}{dx^2}\Big|_{x_0} + \cdots \right]$

$\approx -\dfrac{dU}{dx}\Big|_{x_0} - (x-x_0)\dfrac{d^2U}{dx^2}\Big|_{x_0} + O((x-x_0)^2)$

↑ gives constant force → ← to point x_0 being a stationary point.

If $\left.\frac{dU}{dx}\right|_{x_0} = 0$ & $\left.\frac{d^2U}{dx^2}\right|_{x_0} > 0$ then

~~HARMONIC~~.

w/ $F \approx -(x-x_0)\left.\frac{d^2U}{dx^2}\right|_{x_0}$

$F<0 \quad x>x_0$ } point x_0 is a stable equilibrium point.
$F>0 \quad x<x_0$

Assum $U(x) \approx U(x_0) + \frac{1}{2}(x-x_0)^2 \left.\frac{d^2U}{dx^2}\right|_{x_0}$

Then

$$\bar{x} = \frac{\int e^{-\beta(U(x_0) + \frac{1}{2}(x-x_0)^2 U''(x_0))} \cdot x \, dx}{\int e^{-\beta(U(x_0) + \frac{1}{2}(x-x_0)^2 U''(x_0))} \, dx}$$

$$= \frac{\int_{-\infty}^{\infty} x e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)} \, dx}{\int e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)} \, dx} = \int_{-\infty}^{\infty} (x-x_0) e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)} \, dx + x_0 \int_{-\infty}^{\infty} e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)} \, dx$$

$$\int e^{-\beta}$$

$$= \frac{\int_{-\infty}^{+\infty} (x-x_0)\, e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)}\, dx}{\int_{-\infty}^{\infty} e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)}\, dx} + x_0$$

$$= x_0 \quad \checkmark$$

(c) Ass $U(x) \approx U(x_0) + \frac{1}{2}(x-x_0)^2 U''(x_0) + \frac{1}{6}(x-x_0)^3 U'''(x_0)$

$$\bar{x} = \frac{\int x\, e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)}\, e^{-\beta \frac{1}{6}(x-x_0)^3 U'''(x_0)}\, dx}{\int e^{-\beta(x-x_0)^2 U''(x_0)}\, e^{-\beta \frac{1}{6}(x-x_0)^3 U'''(x_0)}\, dx}$$

$$= x_0 + \frac{\int (x-x_0)\, e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)}\, e^{-\beta \frac{1}{6}(x-x_0)^3 U'''(x_0)}\, dx}{\int e^{-\beta(x-x_0)^2 U''(x_0)}\, e^{-\beta \frac{1}{6}(x-x_0)^3 U'''(x_0)}\, dx}$$

$$= x_0 + \frac{\int x e^{-\frac{\beta}{2}x^2 U''(x_0)} \cdot \left[1 + -\frac{\beta}{6}x^3 U'''(x_0) + O(x^6)\right] dx}{\int e^{-\frac{\beta}{2}x^2 U''(x_0)} \left[1 - \frac{\beta}{6}x^3 U''(x_0) + \frac{\beta^2}{36}x^6 U'''(x_0)^2 + O(x^9)\right]}$$

$e^x = \sum \frac{x^n}{n!}$

$$= x_0 + \frac{\int x e^{-\frac{\beta}{2}x^2 U''(x_0)} dx - \frac{\beta}{6}\int x^4 e^{-\frac{\beta}{2}x^2 U''(x_0)} dx}{\int e^{-\frac{\beta}{2}x^2 U''(x_0)} dx}$$

(the $x e^{-\frac{\beta}{2}x^2 U''(x_0)}$ term integrates to 0)

Now

$$\int e^{-\frac{\beta}{2}x^2 U''(x_0)} dx$$

let $v^2 = \frac{\beta}{2}U''(x_0)x^2$ $\quad v = \sqrt{\frac{\beta U''(x_0)}{2}}\, x$

$$\int e^{-v^2} \frac{\sqrt{2}}{\sqrt{\beta U''(x_0)}} dv \qquad dv = \sqrt{\quad} dx$$

$$= \frac{\sqrt{2\pi}}{\sqrt{\beta U''(x_0)}}$$

Now

$$\int x^4 e^{-\frac{\beta}{2}U''(x_0)x^2}\,dx$$

let $v = \sqrt{\frac{\beta U''(x_0)}{2}}\,x \qquad x = \sqrt{\frac{2}{\beta U''(x_0)}}\,v$

$$= \int \left(\frac{2}{\beta U''(x_0)}\right)^2 v^4 e^{-v^2} \sqrt{\frac{2}{\beta U''(x_0)}}\,dv$$

$$= \left(\frac{2}{\beta U''(x_0)}\right)^{5/2} \underbrace{\int v^4 e^{-v^2}\,dv}$$

$$\frac{3\sqrt{\pi}}{4}$$

$$\therefore \bar{x} = x_0 + \frac{-\frac{\beta}{6}\left(\frac{3\sqrt{\pi}}{4}\right)\left(\frac{2}{\beta U''(x_0)}\right)^{5/2}}{\sqrt{\pi}\left(\frac{2}{\beta U''(x_0)}\right)^{1/2}}$$

$$= x_0 - \frac{\beta}{6}\frac{3}{4}\left(\frac{2}{\beta U''(x_0)}\right)^2 = x_0 - \frac{1}{8}\cdot\frac{4}{\beta\,(U''(x_0))^2}$$

$$= x_0 - C_4 \cdot kT$$

04-13-05

B (1) compute $U''(x_0) = \cdots$

(6.33)

v_{mp} = velocity at which $D(v)$ is largest.

$$= \sqrt{\frac{2kT}{m}}$$

$$M_{O_2} = \frac{2.16\ g}{6.022 \cdot 10^{23}} = \frac{2.16 \cdot 10^{-3}\ kg}{6.022 \cdot 10^{23}}$$

$$kT = (1.381 \cdot 10^{-23}\ J/K)(300\ K) =$$

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}} = \cdots$$

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

(6.34) $$D(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{\frac{-mv^2}{2kT}}$$

$$m_{N_2} = \frac{2.14 \cdot 10^{-3}\ kg}{6.022 \cdot 10^{23}} \cong \cdots \frac{2.14}{6} \cdot 10^{-26}\ kg$$

$kT = (1.381 \cdot 10^{-23}\ J/K)\ T$ = [illegible]

$\frac{m}{kT}$ = spaced in units of $\hat{v} = \sqrt{\frac{kT}{m}}$ sxy

(6.35) $$D(v)dv = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2kT}}\, dv$$

$$\frac{dD}{dv} = \left(\frac{m}{2\pi kT}\right)^{3/2} \cdot 4\pi \left[2v e^{-\frac{mv^2}{2}\cdot\beta} + v^2\left(-\frac{2mv\beta}{2}\right)e^{-\frac{mv^2}{2}\beta}\right] \underset{\text{set}}{=} 0$$

$$= 2v - v^3 m\beta = 0$$

$$v = 0 \quad \text{or} \quad 2 = v^2 m\beta$$

$$v = \sqrt{\frac{2}{m\beta}} = \sqrt{\frac{2kT}{m}} \quad \checkmark$$

(6.36) eq 6.51

$$\bar{v} = \sum_{\text{all } v} v D(v)dv = \int_0^{+\infty} v\, C_4\, 4\pi v^2 e^{-\frac{mv^2}{2}\cdot\beta}\, dv$$

$$= 4\pi C_4 \int_0^{\infty} v^3 e^{-\frac{mv^2}{2}\beta}\, dv \qquad \text{let} \quad u = \sqrt{\frac{m\beta}{2}}\, v$$

$$= 4\pi C_4 \int_0^{\infty} \left(\frac{2}{m\beta}\right)^{3/2} e^{-u^2} u^3 \left(\frac{2}{m\beta}\right)^{1/2} du \qquad dv = \sqrt{\frac{2}{m\beta}}\, du$$

$$= 4\pi q \left(\frac{2}{m\beta}\right)^2 \int_0^\infty v^3 e^{-v^2} dv$$

(6.37)

$$\overline{v^2} = \int_0^\infty v^2 D(v)\,dv = \int_0^\infty v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2}\beta}\,dv$$

$$= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^4 e^{-\frac{mv^2}{2}\beta}\,dv$$

let $u = \sqrt{\frac{m}{2}\beta}\, v \qquad v = \left(\frac{2}{m\beta}\right)^{1/2} u$

$$dv = \left(\frac{2}{m\beta}\right)^{1/2} du$$

$$= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty \left(\frac{2}{m\beta}\right)^{1/2}\left(\frac{2}{m\beta}\right)^{1/2} u^4 e^{-u^2}\,du$$

$$= 4\pi \left(\frac{1}{2\pi}\right)^{3/2} \left(\frac{m}{\beta}\right)^{3/2}$$

$$= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{2}{m\beta}\right)^{5/2} \int_0^\infty u^4 e^{-u^2}\,du$$

(6.38) $f = \int_0^{300\,m/s} D(v)\,dv = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \int_0^{300\,m/s} v^2 e^{-\frac{mv^2}{2}\beta}\,dv$

~~Let~~ $u = \left(\frac{m\beta}{2}\right)^{1/2} v \qquad v =$

$$f = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \int_0^{(\frac{m\beta}{2})^{1/2}\cdot 300} \left(\frac{2}{m\beta}\right)^{2/2}\cdot\left(\frac{2}{m\beta}\right)^{1/2} e^{-u^2} u^2\,du$$

$$= \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \int_0^{\frac{300}{(\frac{2}{m\beta})^{1/2}}}$$

$$= \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \left(\frac{2}{m\beta}\right)^{3/2} \int_0^{\frac{300}{(\frac{2}{m\beta})^{1/2}}} u^2 e^{-u^2}\,du$$

(6.39)

$N_2 =$

$$m_{N_2} = \frac{2.14\cdot 10^{-3}\ kg}{6.022\cdot 10^{23}}$$

$T = 1000\ k.$

$$P = \int_{11\cdot 10^3\ m/s}^{\infty} D(v)\,dv$$

But "several" of these particles will not be pointing in the outward direction

$$P = \int_{11\cdot 10^3\ m/s}^{\infty} \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2kT}}\, dv = \cdots$$

(b) $m_{H_2} = \frac{2\cdot 10^{-3}}{6.02\cdots}$

$m_{He} =$ ——

Based on P_{N_2} / P_{H_2}; P_{He} one should find approximately the same # of molecules.

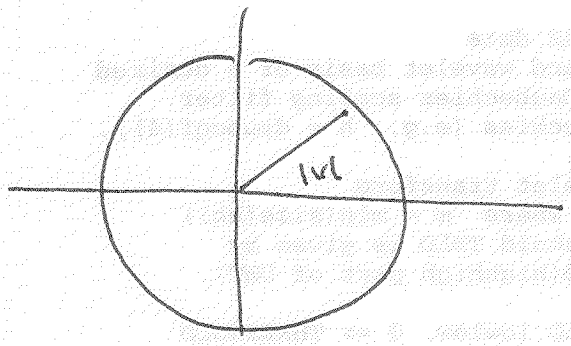
(c)

$$P = \int_{2.4\cdot 10^3\ m/s}^{\infty} D(v)\, dv \approx 1.$$

it is very probable that molecules will be able to escape

6.40 Don't we know? Must be inelastic collisions

6.41



$D(v)$ ∝ (prob molec has vector $\vec{v}$) × (# of vectors $\vec{v}$ w/ $|\vec{v}| = v$)

$$\propto e^{-\frac{mv^2}{2kT}} \cdot 2\pi v$$

$$D(v) = 2\pi C v e^{-\frac{mv^2}{2kT}}$$

$$1 = 2\pi C \int_0^\infty v e^{-\frac{mv^2}{2kT}} dv \qquad \text{solve for } C$$

most likely vector $\vec{v} = 0$ are w/ maximum $D(v)$

most likely speed $\frac{dD(v)}{dv} = 0 \Rightarrow v = \ldots$

6.42 $Z_{h.o.} = \frac{1}{1-e^{-\beta\epsilon}}$

(a) $F = -kT \ln Z$

$F = -kT \ln\left(\frac{1}{1-e^{-\beta\epsilon}}\right)$ is there a power of N to a Nonrelativistic factor of N!?

Think yes

$Z_{total} = \frac{Z_{ho}^{\,N}}{N!}$

$F = -kTN \ln\left(\frac{1}{1-e^{-\beta\epsilon}}\right)$

(b) $S = -\left.\frac{\partial F}{\partial T}\right|_{V,N} = -kN \ln\left(\frac{1}{1-e^{-\beta\epsilon}}\right) - kTN\left(1-e^{-\beta\epsilon}\right)\circ$

$\frac{\partial}{\partial T}\left(1-e^{-\beta\epsilon}\right)$

$= -kN \ln\left(\frac{1}{\quad}\right) - kTN\left(1-e^{-\beta\epsilon}\right)\cdot -\frac{1}{kT^2}\frac{\partial}{\partial\beta}\left(1-e^{-\beta\epsilon}\right)$

$\left\{\frac{\partial\beta}{\partial T} = -\frac{1}{kT^2}\right\}$

6.43

$$S = -k \sum_s P(s) \ln P(s)$$

(a) Isolated system $P(s) = \frac{1}{\Omega}$? Don't see

$$S = -k \sum_s \frac{1}{\Omega} \ln(1/\Omega)$$ know $\sum_s P(s) = 1$

$= \sum_s \frac{\ln(\Omega)}{\Omega}$ can I factor out $\ln(1/\Omega)$ to get

$$S = -k \ln(1/\Omega) \sum_s 1/\Omega = +k \ln \Omega \checkmark$$

(b) $P(s) = \frac{e^{-E(s)\beta}}{Z}$ $\sum_s P(s) = 1$

$$S = -k \sum \frac{e^{-E(s)\beta}}{Z} \ln\left(\frac{e^{-E(s)\beta}}{Z}\right)$$

$$= -\frac{k}{Z} \sum e^{-E(s)\beta} \ln\left(\frac{e^{-E(s)\beta}}{Z}\right)$$

How show?

(6.44)

$$Z_{total} = \frac{Z_1^N}{N!}$$

$\{\ln N! \approx N\ln N - N\}$

$$F = -kT\ln(Z_{total}) = -kTN\ln Z_1 + kT\ln N!$$

$$\approx -kTN\ln Z_1 + kTN\ln N - kTN$$

$$= kTN\left[-\ln Z_1 + \ln N - 1\right]$$

$$\mu = \left.\frac{\partial F}{\partial N}\right|_{V,T} = kT\left[-\ln Z_1 + \ln N - 1\right] + kTN\left[\frac{1}{N}\right]$$

$$= kT + kT\left[-\ln Z_1 + \ln N - 1\right]$$

$$= kT\left[-\ln Z_1 + \ln N\right]$$

(6.45)

$$S = -\left.\frac{\partial F}{\partial T}\right|_{V,N}$$

$$F = -NkT\left[\ln V - \ln N - \ln V_Q + 1\right] + F_{int}$$

$$\left.\frac{\partial F}{\partial T}\right|_{V,N} = -Nk\left[\ln V - \ln N - \ln V_Q + 1\right] - NkT\left[-\frac{1}{V_Q}\frac{\partial V_Q}{\partial T}\right] + \left.\frac{\partial F_{int}}{\partial T}\right|_{V,N}$$

$$\frac{1}{V_Q}\frac{\partial V_Q}{\partial T} = \left(-\frac{3}{2}\frac{V_Q}{T}\right)\frac{1}{V_Q} = -\frac{3}{2}\cdot\frac{1}{T}$$

$$V_Q = \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3 = \left(\frac{h}{\sqrt{2\pi m k}}\right)^3 \cdot T^{-3/2}$$

$f = x^p$

$\frac{df}{f} = p\frac{dx}{x}$

$$\therefore \quad S = -\left.\frac{\partial F}{\partial T}\right|_{V,N} = +Nk\left[\ln V - \ln N - \ln V_Q + 1\right] + NkT\left(\frac{3}{2}\cdot\frac{1}{T}\right) + -\left.\frac{\partial F_{int}}{\partial T}\right|_{V,N}$$

$$\Rightarrow \quad S = +Nk\left[\ln V - \ln N - \ln V_Q + 1 + \frac{3}{2}\right] - \left.\frac{\partial F_{int}}{\partial T}\right|_{V,N}$$

Q-6403 ✓

$$S = +Nk\left[\ln\left(\frac{V}{V_Q \cdot N}\right) + \frac{5}{2} \right] = \frac{\partial F_{int}}{\partial T}\Big|_{V,N} \qquad \text{eq } 6.92 \checkmark$$

$$= Nk \ln($$

$$\mu = \frac{\partial F}{\partial N}\Big|_{T,V} = \frac{\partial}{\partial N}\Big|_{T,V}\left[-NkT\left[\ln V - \ln N - \ln V_Q + 1\right] + F_{int} \right]$$

$$= -kT\left[\ln V - \ln N - \ln V_Q + 1\right] - NkT\left[-\frac{1}{N}\right] + \frac{\partial F_{int}}{\partial N}\Big|_{T,V}$$

$$= kT\left[\ln\left(\frac{NV_Q}{V}\right)\right] + \frac{\partial F_{int}}{\partial N}\Big|_{T,V}$$

$$F_{int} = -kT \ln Z$$

$$\frac{\partial F_{int}}{\partial N} = -\frac{kT}{Z}\frac{\partial Z_{int}}{\partial N} \qquad Z = \frac{Z_{int}^{N}}{N!}$$

$$= -kT\left[\ln\left(\frac{V}{NV_Q}\right)\right] + \frac{\partial}{\partial N}(-kT \ln Z_{int})$$

$$= -kT\left[\ln\left(\frac{V}{NV_Q}\right) + \frac{\partial(N \ln Z_{int})}{\partial N}\right]$$

w/ Z_{int} independent of N.

$$\Rightarrow \mu = -kT\left[\ln\left(\frac{VZ_{int}}{NV_Q}\right)\right] \qquad \text{eq } 6.93 \checkmark$$

6.46 $\dfrac{VZ_{int}}{NV_Q}$

$V \sim 1$ liter $= 10^{-3}\ m^3$

$N \sim 6.23 \cdot 10^{23}$

$V_Q \sim (2\cdot 10^{-11})^3 \sim 8\cdot 10^{-33}$

Nitrogen

$Z_{int} \sim$ Don't know. what values this has? $\gg 1$

$$\frac{(10^{-3})(?)}{(10^{23})(10^{-33})} \gg 1$$

6.47 $Z_{1d} = \dfrac{L}{l_Q} \qquad l_Q = \sqrt{\dfrac{2\pi m k T}{h^2}}$

Assum we have translat. at translational degrees of freedom would be when

$Z_{1d} \sim 1$ or $Z_{1d} \ll 1$

$\Rightarrow L \sim l_Q \Rightarrow \sqrt{\dfrac{2\pi m k T}{h^2}} = L$

$$\frac{2\pi m k T}{h^2} = L^2$$

$$\Rightarrow T = \frac{h^2 L^2}{2\pi m k} \rightarrow T = \frac{(6.626\cdot 10^{-34}\ J\cdot s)^2 (10^{-2}\ m)^2}{2\pi \left(\frac{14\cdot 10^{-3}\ kg}{6.022\cdot 10^{23}}\right)(1.381\cdot 10^{-23}\ J/k)}$$

$$\Rightarrow T = 21\cdot 10^{-23}\ k.$$

(6.48) $Z_{int} = Z_e \cdot Z_{vot}$

$$F_{int} = -kTN \ln Z_{int} = -kTN \ln(Z_e Z_{vot})$$

From eq 6.92 in text

$$S = Nk\left[\ln\left(\frac{V}{NV_Q}\right) + \frac{5}{2}\right] - \frac{\partial}{\partial T}\left(-kTN \ln(Z_e Z_{vot})\right)$$

~~$kN \ln(Z_e Z_{vot})$~~

$= $ ~~$\ln\left(\frac{V Z_e Z_{vot}}{NV_Q}\right)$~~ w/ $Z_{vot} \cong \frac{kT}{\epsilon}$

$$\Rightarrow S = Nk\left[\ln\left(\frac{V}{NV_Q}\right) + \frac{5}{2}\right] + kN \ln(Z_e Z_{vot})$$

$$+ \frac{kNT}{Z_e Z_{vot}} \cdot Z_e \cdot \frac{k}{\epsilon}$$

$$\Rightarrow S = Nk\left[\ln\left(\frac{V z_t z_{rot}}{N V_Q}\right) + \frac{5}{2}\right] + kN$$

$$= Nk\left[\ln\left(\frac{V z_t z_{rot}}{N V_Q}\right) + \frac{7}{2}\right]$$

~~Eqn~~ ~~$PV = NkT$~~ ~~$V = kT$~~ ~~$Nk = PV$~~

~~(XXXXXXXXXX #~~

~~$S =$~~ 1 mole $\Rightarrow N = N_A = 6.022 \cdot 10^{23}$

Also $PV = NkT$ ~~$\div N$~~ $\Rightarrow \frac{V}{N} = \frac{kT}{P}$

$z_{rot} = \frac{kT}{\epsilon}$ $\quad t_Q \cong 0.00018$ Å (pg 236 Schroeder) ✓

(b) $\mu = -kT \ln\left(\frac{V z_t z_{rot}}{N V_Q}\right)$ ...

(6.49) U, H, F, G, S, μ $\quad \epsilon_{N_2} = .0025\ eV$

$T = 273\ k$

$P = 1\ atm$

... Not sure

(6.50) $G = U - TS + PV \qquad dU = TdS - pdV$

So $G = U_{int} + \frac{3}{2}NkT - T\left(Nk\left[\ln\left(\frac{V}{N v_Q}\right) + \frac{5}{2}\right] - \frac{\partial F_{int}}{\partial T}\right) + PV$

$\overset{?}{=} N\left(-kT \ln\left(\frac{V}{N v_Q} Z_{int}\right)\right)$

what is Z_{int}?

?

(6.51) $Z_{tr} = \frac{1}{h^3}\int d^3r\, d^3p\, e^{-E_{tr}/kT} \qquad E_{tr} = +\left(\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}\right)$

$= \frac{1}{2m}\left(p_x^2 + p_y^2 + p_z^2\right)$

$= \frac{1}{h^3}\int d^3r \cdot \int d^3p\, e^{-E_{tr}/kT}$

$\int d^3r = V$

$= \frac{V}{h^3}\int d^3p \exp\left\{-\frac{1}{2m(kT)}\left(p_x^2 + p_y^2 + p_z^2\right)\right\}$

$= \frac{V}{h^3}\prod_{i=1}^{3}\int dp\, e^{-\frac{p^2}{2mkT}}$

let $v^2 = \frac{p^2}{2mkT} \Rightarrow v = \frac{p}{\sqrt{2mkT}}$

$\Rightarrow p = \sqrt{2mkT}\, v.$

$dp = \sqrt{2mkT}\, dv$

so $Z_{tr} = \frac{V}{h^3}\left(\sqrt{2mkT}\int_{-\infty}^{\infty} dv\, e^{-v^2}\right)^3$

$= \frac{V}{h^3}(2mkT)^{3/2}\left(\sqrt{\pi}\right)^3$

$= V \cdot \left(\frac{2\pi mkT}{h^2}\right)^{3/2} = \frac{V}{\left(\frac{h^2}{2\pi mkT}\right)^{3/2}}$

defin $v_Q = \left(\frac{h^2}{2\pi mkT}\right)^{3/2}$ ✓.

(6.52) $E = pc$ vs. $E = \frac{p^2}{2m}$

$$Z_1 = \frac{1}{h}\int dx\,dp\, e^{-E/kT} = \frac{1}{h}\cdot L\int_{-\infty}^{+\infty} dp\, e^{-p/ckT}$$

$v = \frac{p}{ckT}$

$dp = ckT\,dv$

$$= \frac{L}{h}\cdot ckT\int_{-\infty}^{\infty} dv\, e^{-v}\cdot$$

$$= \frac{L\cdot ckT}{h}\left(-e^{-v}\Big|_{-\infty}^{+\infty}\right) \quad X$$

in $E = pc$ p is $|p|$

$$\therefore Z_1 = \frac{L}{h}\int_0^{\infty} \cdots = \frac{L\cdot ckT}{h}\left(-e^{-v}\Big|_0^{\infty}\right)$$

$$= -\frac{L\,ckT}{h}(0-1) = \frac{L\,ckT}{h}$$

$$= \frac{L}{\left(\frac{h}{ckT}\right)} \quad \therefore \quad l_Q = \frac{h}{ckT}$$

Check units $[l_Q] = \dfrac{J\cdot s}{\left(\frac{m}{s}\right)\left(\frac{J}{J}\right)(\not{K})} = \dfrac{s^2}{m}$? error somewhere

6.53

$H_2 \rightleftharpoons 2H$

$$K = \frac{[H]^2}{[H_2]} \cong \frac{P_H^2}{P_{H_2} \cdot P^\circ}$$

P's = partial pressures

... Not sure.

(7.1) $Z = \ldots$?

$Z = 1 + e^{-(\epsilon-\mu)/kT}$

$P = \dfrac{e^{-(\epsilon-\mu)/kT}}{1+e^{-(\epsilon-\mu)/kT}}$ w/ $\mu = \mu(P^\circ_{O_2})$

$\epsilon = -.7\,eV.$

$PV = NkT$ so $\mu(P^\circ) = -kT \ln\left(\dfrac{Z_{int}\, kT}{V_a P^\circ}\right)$

plot $P(P^\circ) = \ldots$

(7.2) $Z = 1 + 2e^{-(-.55eV - \mu(P_{O_2}))/kT} + e^{-(-1.3eV - \mu(P_{O_2}))/kT}$

Then

$P_{\text{two sites occupied}}(P_{O_2}) = \dfrac{1}{Z(P_{O_2})}$

$P_{\text{one site occupied}}(P_{O_2}) = \dfrac{2e^{-(-.55eV - \cdots)/kT}}{Z(P_{O_2})}$

$P_{\text{two sites occ}} = \dfrac{e^{-(1.3eV \cdots)/kT}}{Z(P_{O_2})}$

(7.3) $Z = 1 + e^{-(\epsilon - \mu)/kT}$

ϵ = ionization energy for H atom

μ = chemical potential of ideal ~~gas~~ gas

(7.4) $Z = 1 + e^{-(\epsilon' - \mu)/kT} + e^{-(\epsilon'' - \mu)/kT}$

How change chemical potential of gas?

(7.5) (a) $P_{\text{ionized}} = 1 - P_{\text{~~site occupied~~ original donor site occupied}} = 1 - \dfrac{2e^{-(E - \mu)/kT}}{1 + 2e^{-(E - \mu)/kT}}$

↑ 2 for 2 electron spin states

(don't change the ionization energy)

(b) $\mu = -kT \ln\left(\dfrac{Z_{int}}{V_Q} \cdot \dfrac{V}{N}\right)$

(c) ?

(d) P . .

(7.6) $$p(s) = \frac{e^{-(E(s) - \mu N(s))/kT}}{Z}$$

$$\frac{\partial p}{\partial N} = 0 \quad \text{solve for } N$$

$$p(s) \cdot \frac{\mu}{kT} - \frac{p(s)}{Z} \frac{\partial Z}{\partial N} = 0$$

$$\Rightarrow \frac{\mu}{kT} - \frac{1}{Z} \frac{\partial Z}{\partial N} = 0$$

$$\sigma_N^2 = \langle N^2 \rangle - \langle N \rangle^2 = \frac{(kT)^2}{Z} \frac{\partial^2 Z}{\partial \mu^2} - \frac{kT}{Z} \frac{\partial Z}{\partial \mu}$$

$$=$$

...

(7.7) Prob 5.23 is $\Phi = U - TS - \mu N$

$F = -kT \ln Z \qquad \Phi = -kT \ln Z$

Show $\Phi = U - TS - \mu N$ + $\Phi = -kT \ln Z$ satisfy same

D.E. w/ initial conditions.

(7.8)

$N_s = 10$.

(a) $Z = \sum_s e^{-\beta E(s)} = \sum_{s=1}^{10} e^{-\beta E_s} = 10$ since each term is 1

(b) $Z = 10^2$

(c) Bosons: can occupy the same state & identical

$\tilde{A}$ $\tilde{B}$
x x

$$Z = \frac{(Z_1)(Z_1)}{2} = \frac{Z_1^2}{2}$$

(d) Fermions: cannot occupy the same state.

$$Z = \frac{Z_1(Z_1 - 1)}{2}$$

(e) $Z = \frac{1}{2!} Z_1^2$

(f) $P_{Fermions} = 0$ can't happen

$$P_{Bosons} = \frac{?}{\left(\frac{Z_1^2}{2}\right)}$$

(7.9) N_2

$$V_Q = \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3$$

$$m_{H_2} = \frac{28\ g/mole\ (14)}{6.022 \cdot 10^{23}\ molecules/mole}$$

$$PV = N \cdot kT$$

$$\frac{V}{N} = \frac{kT}{P}$$

$\frac{V}{N} \gg V_Q$ Boltzmann statistics regime.

$\frac{V}{N} = \frac{kT}{P}$ $\quad V_Q = \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3$

$$\frac{kT}{P} \gg \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3$$

$$(2\pi m k T)^{3/2} \cdot kT \gg h^3 P$$

Density constant $\Rightarrow \frac{1}{V} = \rho_N$

$$\Rightarrow \left(\frac{N}{V}\right) = \frac{P}{kT}$$

$$(2\pi m k T)^{3/2} \gg h^3 \left(\frac{P}{kT}\right)$$

7.10

(a)

	Identical Fermions	Identical Bosons	Distinguishable
E_{gnd}	$\epsilon_1+\epsilon_2+\epsilon_3$	$3\epsilon_1$	$\epsilon_1+\epsilon_2+\epsilon_3$ vs. $3\epsilon_1$

(b) ϵ_1 ?

(c) ?

(d) ?

(7.11) $$\bar{n}_{FD} = \frac{1}{e^{(\epsilon-\mu)/kT} + 1}$$

(a) $\epsilon - \mu = -.1$ eV

$kT = (8.617 \cdot 10^{-5}\ eV/K)(300K)$

$\approx 24 \cdot 10^{-3}$ eV

$\approx 2.4 \cdot 10^{-2}$ eV

(b) $\epsilon - \mu = -.01$ eV

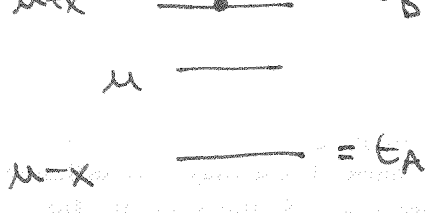
(c) $\epsilon - \mu = 0$

$\bar{n}_{FD} = \frac{1}{2}$

(d) $\epsilon - \mu = .01$ eV

(e) $\epsilon - \mu = 1$ eV

(7.12)



$$P_{\text{B occupied}} = \frac{e^{-(\epsilon_B-\mu)/kT} + 0}{e^{-(\epsilon_B-\mu)/kT} + 1 + e^{-(\epsilon_A-\mu)/kT}} = \frac{e^{-x/kT}}{1 + e^{-x/kT} + e^{+x/kT}}$$

$$P_{A\ \text{unoccupied}} = \frac{e^{-(\epsilon_B-\mu)/kT} + 1}{e^{-x/kT} + e^{+x/kT} + 1} = \frac{e^{-x/kT} + 1}{e^{-x/kT} + e^{+x/kT} + 1} \quad ?$$

(7.13) $$\bar{n}_{BE} = \frac{1}{e^{(\epsilon-\mu)/kT} - 1} \quad \text{or} \quad z = \frac{1}{1 - e^{-(\epsilon-\mu)/kT}} \quad \text{Bosons}$$

(a) $\epsilon - \mu = .001$ eV

(b) $\epsilon - \mu = 0.01$ eV

(c) $\epsilon - \mu = .1$ eV

(d) $\epsilon - \mu = 1$ eV

$$P = \frac{1}{Z} e^{-(n\epsilon - n\mu)/kT} = \frac{e^{-n(\epsilon-\mu)/kT}}{Z} \qquad n = 0, 1, 2, 3.$$

(7.14) For large energy $\{\epsilon - \mu\}$ all statistics should obey Boltzmen statistics or expect near about the Boltzmen distribution

$$\bar{n}_{Boltzmen} = e^{-(\epsilon-\mu)/kT}$$

$$\bar{n}_{BE} = \frac{e^{-(\epsilon-\mu)/kT}}{1 - e^{-(\epsilon-\mu)/kT}} = e^{-(\epsilon-\mu)/kT} \sum_{k \geq 0} e^{-((\epsilon-\mu)/kT)\cdot k}$$

$$\bar{n}_{BE} \cong e^{-(\epsilon-\mu)/kT}\left[1 + e^{-(\epsilon-\mu)/kT} + H.O.T.\right]$$

$$\bar{n}_{FD} = \frac{e^{-(\epsilon-\mu)/kT}}{1+e^{-(\epsilon-\mu)/kT}} = e^{-(\epsilon-\mu)/kT}\sum_{n\geq 0}(-1)^n e^{-(\epsilon-\mu)n/kT}$$

$$\cong e^{-(\epsilon-\mu)/kT}\left[1 - e^{-(\epsilon-\mu)/kT}\right]$$

∴ Both statistics have relative error of

$$\left| e^{-(\epsilon-\mu)/kT} \right| \ll .01$$

$$\Rightarrow \quad -\frac{(\epsilon-\mu)}{kT} \ll \ln(10^{-2})$$

$$(\epsilon-\mu) \gg \underline{2kT\ln(10)}$$

How tight violated?

7.15

7.31 $\quad \bar{n}_{\text{Boltzmann}} = e^{-(\epsilon-\mu)/kT}$

~~$\sum e^{-(\epsilon-\mu)/kT}$~~ $\quad \sum_{n\geq 0} e^{-(\epsilon-\mu)n/kT} = N$ $\quad$ why? do it so?

$$= \sum_{n\geq 0} e^{-\frac{\epsilon n}{kT}} \cdot \sum_{n\geq 0} e^{+\mu n/kT} = N$$

$Z_1 \quad = \quad \frac{1}{1-e^{\mu/kT}}$

$$\rightarrow \quad \frac{Z_1}{N} = 1 - e^{\mu/kT}$$

$$e^{\mu/kT} = 1 - \frac{Z_1}{N}$$

$$\mu = kT \ln\left(1 - \frac{Z_1}{N}\right) \quad ?$$

7.16

(a) ?

(b) ?

(c) ?

(d)

7.17 ? $q=0$ $\quad q=1$

↑

Energy

● = All in the ground state

(7.18)

$$Z = e^{-0(\epsilon-\mu)/kT} + e^{-1(\epsilon-\mu)/kT} + e^{-2(\epsilon-\mu)/kT}$$

$$\bar{n} = \sum_n n P(n) = 0 \cdot P(0) + 1 \cdot P(1) + 2P(2)$$

$$= \frac{e^{-(\epsilon-\mu)/kT}}{Z} + \frac{e^{-2(\epsilon-\mu)/kT}}{Z}$$

plot picture like Fig. 7.6.

(7.19)

$\rho_{Cu} = ?$

$m_{Cu} = 63.546 \text{ g/mol}$ $\qquad V = 7.12 \text{ cm}^3/\text{mol}$

$$\rho_{Cu} = \frac{63.546 \text{ g/mol}}{7.12 \text{ cm}^3/\text{mol}} \approx 9 \text{ g/cm}^3$$

$$\varepsilon_F = \frac{h^2}{8m}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3} = \cdots$$

$$m = \frac{63.546 \text{ g/mol}}{6.022 \cdot 10^{23}} = \cdots$$

$$T_F = \frac{\varepsilon_F}{k} = \cdots$$

$$P = -\frac{\partial}{\partial V}\left[\frac{3}{5}N \cdot \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}\right] = \frac{2N\varepsilon_F}{5V} = \cdots$$ degeneracy pressure

$$B = -V\left.\frac{\partial P}{\partial V}\right|_T = \frac{10}{9}\frac{U}{V} =$$

$$= -V\frac{\partial}{\partial V}\left[\frac{3}{5}N\frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}\right] = \cdots$$

(7.20) [illegible] $D = 10^{32}/m^3$

To determine the statistics to use in dealing w/ this system

consider $\frac{V}{N} = D^{-1} = 10^{-32}\ m$

V.S. $V_Q = \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3 = \left(\frac{(6.626\cdot 10^{-34}\ J\cdot s)}{\sqrt{2\pi(9.109\cdot 10^{-31}\ kg)(1.381\cdot 10^{-23}\ J/K)([illegible])10^7\ K)}}\right)^3$

[illegible]

$V_Q = 1.309\cdot 10^{-32}$

since $V_Q \sim \frac{V}{N}$ I would argue that ⇒ use degenerate Fermi gas.

(7.21) $\frac{N}{V} = .18\ (10^{-15}\ m)^{-3} = (.18)\,10^{45}\ m^{-3}$

$\varepsilon_F = \frac{h^2}{8m}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3}$

Since we can hold 4 waves this

$N = \underline{4} \times (\text{vol of } \tfrac{1}{8} \text{ sphere}) = 4\cdot\left(\frac{1}{8}\right)\frac{4}{3}\pi n_{max}^3 = \frac{2}{3}\pi n_{max}^3$

Also $\varepsilon_F = \frac{h^2 n_{max}^2}{8mL^2}$ ↓ $n_{max} = \left(\frac{3}{2}\frac{N}{\pi}\right)^{1/3}$

So that

$$\varepsilon_F = \frac{h^2}{8mL^2}\left(\frac{3}{2}\frac{N}{\pi}\right)^{2/3}$$

$$= \frac{h^2}{8mV^{2/3}}\left(\frac{3}{2}\frac{N}{\pi}\right)^{2/3}$$

$$T_F = \frac{\varepsilon_F}{k}$$

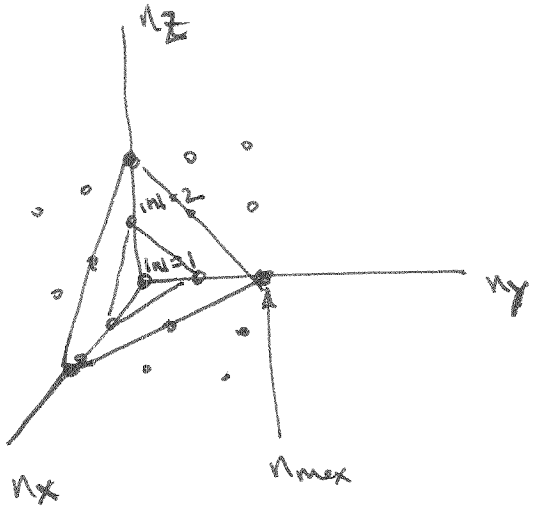
(7.22) $\epsilon = pc = \frac{hc}{\lambda}$ $\qquad \lambda_n = \frac{2L}{n}$

$\therefore \epsilon_n = c\frac{h}{2L}n$ $\qquad p_n = \frac{h}{\lambda_n} = \frac{h\cdot n}{2L}$

w/ 3 degrees of most freedom

$$\epsilon_{\vec{n}} = \frac{ch}{2L}(n_x + n_y + n_z)$$

$$\therefore \varepsilon_F = \frac{ch}{2L}(n_{max})$$



N = Vol of pyramid w/ side of length n_{max}

= Vol of tetrahedron w/ side of length n_{max} $\sim \left(\frac{1}{2}hb\right)\cdot h \sim O(n_{max}^3)$

$= C_t n_{max}^3$

$$\varepsilon_F = \frac{ch}{2L}\left(\frac{N}{c_t}\right)^{1/3} = \frac{k_F^2}{2m}$$

$$= \frac{hc}{c_t^{1/3}}\left(\frac{N}{V}\right)^{1/3} \qquad \text{with} \quad c_t = \left(\frac{3}{8\pi}\right)$$

(b) $$U = 1\sum_{n_x}\sum_{n_y}\sum_{n_z}\epsilon(\vec{n}) = \sum_{n_x}\sum_{n_y}\sum_{n_z}\frac{ch}{2L}(n_x+n_y+n_z)$$

$$= \int_0^{n_{max}}\int_0^{n_{max}-n_x}\int_0^{n_{max}-n_x-n_y}\frac{ch}{2L}(n_x+n_y+n_z)\,dn_z\,dn_y\,dn_x$$

$$= \frac{ch}{2L}\int_0^{n_{max}}\int_0^{n_{max}-n_x}\left(n_x n_z + n_y n_z + \frac{n_z^2}{2}\right)\Bigg|_0^{n_{max}-n_x-n_y} dn_y\,dn_x$$

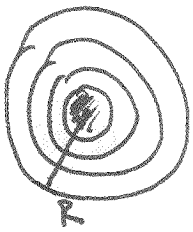
$$= \frac{ch}{2L}\int_0^{n_{max}}\int_0^{n_{max}-n_x}\Big[n_x(n_{max}-n_x-n_y) + n_y(n_{max}-n_x-n_y) \cdot$$

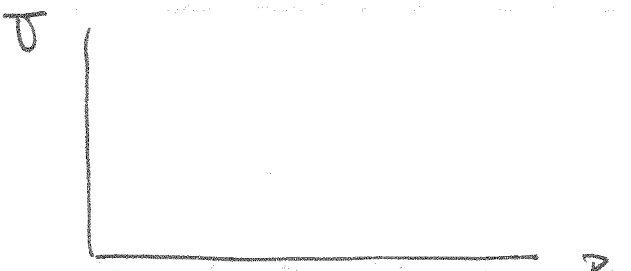
$$+ \frac{1}{2}(n_{max}-n_x-n_y)^2\Big]\,dn_y\,dn_x$$

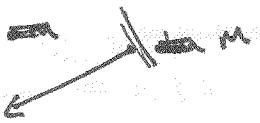
$$= \frac{ch}{2L}\int_0^{n_{max}}\int_0^{n_{max}-n_x} \quad \cdots$$

7.23

(a) $U_{grav} = -C\frac{GM^2}{R}$

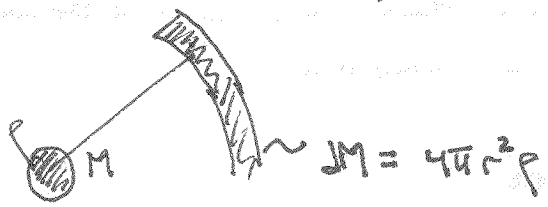






to bring dm to mass m from ∞ requires $W = \int F\,dr$

$F = \frac{GM \cdot dm}{r^2}$ $\qquad F = \frac{GMm}{r^2}$



$M = \frac{4}{3}\pi r^3 \rho$

(b) ?

(c) $U_{total} = U_{grav} + U_{I.E} = -C_1\frac{GM^2}{R} + \frac{C_2 h^2 M^{5/3}}{m_e m_p^{5/3} R^2}$

$U_{total}(R)$

$\frac{dU_{total}}{dR} = C_1\frac{GM^2}{R^2} - \frac{2C_2 h^2 M^{5/3}}{m_e m_p^{5/3} R^3} = 0 \Rightarrow R =$

(d) $R = \ldots$

$$\rho = \frac{M}{\frac{4\pi}{3}R^3} = \ldots$$

(e) $\epsilon_F = \frac{U}{(\frac{3}{5}N)}$

$$\epsilon_F = \frac{\hbar^2}{2m}\left(\frac{3N}{\pi V}\right)^{2/3} \qquad kT \ll \epsilon_F \quad ?$$

(f) ?

(g) $\langle KE \rangle \cong mc^2$

$\langle KE \rangle =$

?

7.24 one - solar - mass all neutron star.

Follow results of the class

7.25

$$C_V = \frac{\pi^2}{2} N k \frac{kT}{\epsilon_F}$$

$N = 6.022 \cdot 10^{23}$

$k = \ldots$

$T = 300k$

$$\epsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$$

$m_{Cu} = (63.54\ g / 6.028 \cdot 10^{23})$

$V = 7.12\ cm^3$

What is the contribution due to lattice vibrations?

7.26 ^3He

(a) $$\epsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$$

Assuming 1) e^- has no mass comp w/ proton
2) proton + neutron have the same mass.

$V = 37\ cm^3$ $\quad m_{^3He} = \dfrac{4.002 + 1.007}{6.022 \cdot 10^{23}}$

$N = 6.022 \cdot 10^{23}$

$$T_F = \frac{\epsilon_F}{k}$$

(b) $C_V = \frac{\pi^2}{2} \frac{N k^2 T}{\epsilon_F}$

(c) 2 states $\forall$ nucleon. $\Omega(N) =$ $\binom{2N}{N}$

=

$S = k \ln \Omega(N)$

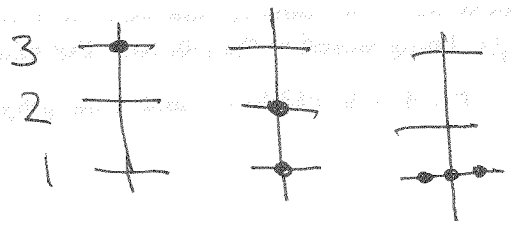
? Not sure

7.27

(a) $q = 3$

$= 2+1$

$= 1+1+1$



?

$p(q)$ = unrestricted partitions

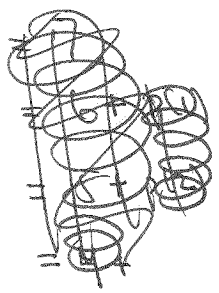
$p(1) = 1$ $p(3) = 3$

$p(2) = 2$ $p(4) = 4$

$p(5) = 7$

(b) $p(7) = ?$

$$p(n) = 1 + \sum_{l=1}^{n-1} (l + p(l))$$



$= 7$

$= 6+1$

$= 5+2,\ 5+1+1$

$= 4+3,\ 4+2+1,\ 4+1+1+1$

$= 3+$

4

~~3+1~~

2+1+1

1+1+1+1

~~2+5~~

5, 4+1, 3+2, 3+1+1

2+2+1, 1+1+1+1+1

2+1+1+1

$p(7) = ?$

7

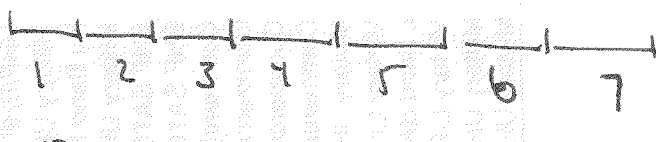
6+1

5+2, 5+1+1

4+3, 4+2+1, 4+1+1+1

3+4, 3+1+3, 3+2+2, 3+1+1+2, 3+1+1+1+1

2+5, 2+1+4, 2+2+3,



0 0 0 0 0 0 1.

1 0 0 0 0 1 0.

0 1 0 0 1 0 0.

2 0 0 0 1 0 0.

0 0 1 1 0 0 0

1 1 0 1 0 0 0

3 0 0 1 0 0 0

0 2 1 0 0 0 0

2 1 1 0 0 0 0

4 2 0 1 0 0 0 0.

1 0 2 0 0 0 0 0.

6 1 0 0 0 0 0

3 2 0 0 0 0 0

1 3 0 0 0 0 0

7 0 0 0 0 0 0

$p(7) = 15$.

(c) ?

(d) $p(q) \approx \dfrac{e^{\pi\sqrt{2q/3}}}{4\sqrt{3}\,q}$

$p(10) =$

$S = k \ln \Omega = k \ln p(q) = \cdots$

Where does temperature come in?

7.28

(a) 2D:

$$\epsilon = \frac{|p|^2}{2m} \qquad p_x = \frac{h}{\lambda_x} \qquad p_y = \frac{h}{\lambda_y}$$

$$\lambda_x = \frac{2L_x}{n} \qquad \lambda_y = \frac{2L_y}{n}$$

$$\therefore \epsilon = \frac{(p_x^2 + p_y^2)}{2m}$$

$$= \frac{h^2\left(\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2}\right)}{2m} = \frac{h^2}{2m}\left(\frac{n_x^2}{4L^2} + \frac{n_y^2}{4L^2}\right) = \frac{h^2}{8mL^2} n^2$$

N = 2× Area of ¼ circle of radius n_{max} $= 2 \cdot \frac{\pi n_{max}^2}{4} = \frac{\pi}{2} n_{max}^2$

$$\downarrow$$

$$n_{max} = \left(\frac{2N}{\pi}\right)^{1/2}$$

$$\epsilon_F = \frac{h^2}{8mA} n_{max}^2$$

$$\rightarrow \epsilon_F = \left(\frac{h^2}{8mA}\right)\left(\frac{2N}{\pi}\right) = \frac{h^2}{4\pi m}\left(\frac{N}{A}\right)$$

~~Avg energy~~

$$U = 2\iint \epsilon(\vec{n})\, dn_x\, dn_y = 2\int_0^{n_{max}} \int_0^{\pi/2} \left(\frac{h^2}{8mA}\right) r^2 \, r\, dr\, d\theta$$

$$\epsilon(\vec{n}) = \frac{h^2}{8mA} |\vec{n}|^2$$

$$= 2\left(\frac{h^2}{8mA}\right)\left(\frac{\pi}{2}\right)\int_0^{n_{max}} r^3\, dr$$

$$= \pi\left(\frac{h^2}{8mA}\right)\frac{n_{max}^4}{4} = \frac{\pi}{4}\left(\frac{h^2}{8mA}\right) n_{max}^4$$

$$\Rightarrow \quad U = \frac{\pi}{4}\left(\frac{\hbar^2}{8mA}\right) n_{max}^2 \cdot n_{max}^2$$

$$= \frac{\pi}{4} \cdot \epsilon_F \left(\frac{2}{\pi} N\right) = \frac{\epsilon_F \cdot N}{2}$$

$$\Rightarrow \quad \frac{U}{N} = \frac{\epsilon_F}{2} \qquad \checkmark.$$

(b) Consider

$$U = 2\int_0^{n_{max}} \int_0^{\pi/2} \epsilon(n)\, n\, dn\, d\theta = \pi \int_0^{n_{max}} \epsilon(n)\, n\, dn$$

But $\epsilon(n) = \frac{\hbar^2}{8mA} n^2 \Rightarrow n = \left(\frac{8mA}{\hbar^2}\right)^{1/2} \epsilon^{1/2}$

$$dn = \left(\frac{8mA}{\hbar^2}\right)^{1/2} \frac{1}{2} \epsilon^{-1/2}\, d\epsilon$$

Thus

$$U = \pi \int_0^{\epsilon_F} \epsilon \left(\frac{8mA}{\hbar^2}\right)^{1/2} \epsilon^{1/2} \cdot \left(\frac{8mA}{\hbar^2}\right)^{1/2} \cdot \frac{1}{2} \cdot \epsilon^{-1/2}\, d\epsilon$$

$$= \frac{\pi}{2}\left(\frac{8mA}{\hbar^2}\right) \int_0^{\epsilon_F} \epsilon\, d\epsilon = \frac{\pi}{2}\left(\frac{8mA}{\hbar^2}\right) \frac{\epsilon_F^2}{2}$$

$$U = \frac{\epsilon_F}{2} \cdot \left(\pi \left(\frac{8mA}{h^2} \right) \epsilon_F \right)$$

From $N = \frac{\pi}{2} n_{max}^2 = \frac{\pi}{2} \cdot \left(\frac{8mA}{h^2} \right) \epsilon_F$

$\therefore U = \frac{\epsilon_F}{2} (2N)$ ... dt by factor of 2?

Now again

$$U = \pi \int_0^{n_{max}} \epsilon(n) n \, dn = \pi \int_0^{\epsilon_F} \epsilon \cdot \left[\frac{1}{2} \left(\frac{8mA}{h^2} \right) \right] d\epsilon$$

$g(\epsilon) = \frac{1}{2} \left(\frac{8mA}{h^2} \right)$ ind. of ϵ V.

(c) for non zero T

$$N = \int_0^\infty g(\epsilon) \left(e^{\frac{1}{(\epsilon - \mu)/kT} + 1} \right) d\epsilon = \frac{1}{2} \left(\frac{8mA}{h^2} \right) \int_0^\infty e^{\frac{1}{(\epsilon-\mu)/kT} + 1} \, d\epsilon$$

$$= \frac{1}{2} \left(\frac{8mA}{h^2} \right) \int_0^\infty \left(\sum_{n \neq 0} (-1)^n e^{n(\epsilon - \mu)/kT} \right) d\epsilon \quad \times \text{ Not convergent series}$$

$$= \frac{1}{2} \left(\frac{8mA}{h^2} \right) \sum_{n \neq 0} (-1)^n \int_0^\infty e^{n(\epsilon-\mu)/kT} \, d\epsilon$$

$$N = \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) \int_0^\infty \frac{e^{-(\epsilon-\mu)/kT}}{1+e^{-(\epsilon-\mu)/kT}}\, d\epsilon$$

$$= \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) \int_0^\infty e^{-(\epsilon-\mu)\beta} \sum_{n\geq 0} e^{-n(\epsilon-\mu)/kT}\, d\epsilon$$

$$= \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) \sum_{n\geq 0} \int_0^\infty e^{-(n+1)(\epsilon-\mu)\beta}\, d\epsilon$$

$$= \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) \left[\sum_{n\geq 0} \frac{e^{-(n+1)(\epsilon-\mu)\beta}}{-(n+1)\beta} \right]_0^\infty$$

$$= \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) \sum_{n\geq 0} e^{+(n+1)\mu\beta} \int_0^\infty e^{-(n+1)\epsilon\beta}\, d\epsilon$$

$$= \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) \sum_{n\geq 0} e^{+(n+1)\mu\beta} \left[\frac{e^{-(n+1)\epsilon\beta}}{-(n+1)\beta} \right]_0^\infty$$

$$= \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) \sum_{n\geq 0} \frac{e^{(n+1)\mu\beta}}{-(n+1)\beta} (0-1)$$

03-18-03

$$= \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) \sum_{n\geq 0} \frac{e^{(n+1)\mu B}}{(n+1)B} \equiv F(\mu)$$

$$= \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) \sum_{n\geq 0} \int e^{(n+1)\mu B}$$

Then $F'(\mu) = \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) \sum_{n\geq 0} e^{(n+1)\mu B}$

$$= \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) e^{\mu B} \sum_{n\geq 0} e^{n\mu B} \qquad e^{\mu B} < 1$$

$$= \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) e^{\mu B} \left(\frac{1}{1-e^{\mu B}}\right) =$$

$$= \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) \frac{1}{(e^{-\mu B}-1)}$$

$\therefore$ $$F'(\mu) = \frac{C}{(e^{-\mu B}-1)}$$

$$F(\mu) = C' \int \frac{d\mu}{e^{-\mu B}-1} = C \int \frac{e^{\mu B}}{1-e^{\mu B}}\, d\mu$$

$$= -\frac{C}{B} \ln(1-e^{\mu B})$$

$$= -\frac{C}{B} \ln(1-e^{+\mu B})$$

$$N = -\frac{1}{2}\left(\frac{8\pi A}{h^2}\right) kT \ln\left(1 - e^{\mu\beta}\right)$$

$$\ln\left(1 - e^{\mu\beta}\right) = -2\frac{h^2}{8m}\left(\frac{N}{A}\right)\frac{1}{kT} = -\frac{h^2}{4m}\left(\frac{N}{A}\right)\frac{1}{kT}$$

$$\Rightarrow 1 - e^{\mu\beta} = \exp\left\{-\frac{h^2}{4m}\left(\frac{N}{A}\right)\frac{1}{kT}\right\}$$

$$e^{\mu\beta} = 1 - e^{-\frac{h^2}{4m}\left(\frac{N}{A}\right)\frac{1}{kT}}$$

$$\Rightarrow \mu = \frac{1}{\beta}\ln\left(1 - e^{-\frac{h^2}{4m}\left(\frac{N}{A}\right)\frac{1}{kT}}\right) = kT \ln(1 - \cdots)$$

(e) $kT \gg \epsilon_F \qquad \epsilon_F = \frac{h^2}{4\pi m}\left(\frac{N}{A}\right)$

$$\therefore \mu = \frac{1}{\beta}\ln\left(1 - e^{-\pi\epsilon_F/kT}\right)$$

Assuming $\epsilon_F \ll kT$

$$\ln(1-x) \cong x - \frac{x^2}{2} - \frac{x^3}{3}$$

$$\ln(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots$$

$$\mu = \frac{1}{\beta}\left[\ln\left(1 - \left(1 - \frac{\pi\epsilon_F}{kT}\right)\right)\right]$$

$$= \frac{1}{\beta}\ln\left(\frac{\pi\epsilon_F}{kT}\right)$$

$$\mu = -kT \ln\left(\frac{kT}{\pi\left(\frac{h^2}{4\pi m}\left(\frac{N}{A}\right)\right)}\right)$$

$$= -kT \ln\left(\frac{A}{N} \cdot \left(\frac{4\pi m kT}{h^2}\right)^{1}\right)$$

similar to an ideal gas.

(7.29)

eq 7.68 is

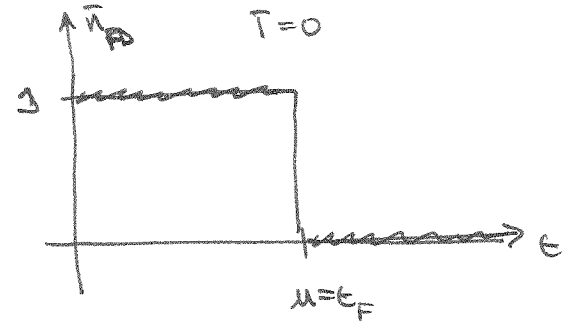
$$U = \frac{3}{5} N \frac{\mu^{5/2}}{\epsilon_F^{3/2}} + \frac{3\pi^2}{8} N \frac{(kT)^2}{\epsilon_F} + \cdots$$

eq 7.54 is

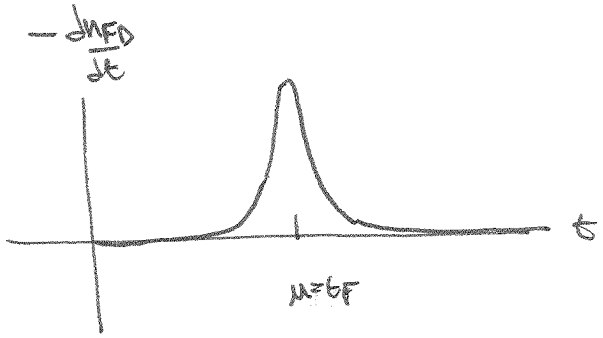
$$U = \int_0^\infty \epsilon g(\epsilon) \bar{n}_{FD}(\epsilon)\, d\epsilon$$

let $g(\epsilon) = g_0 \epsilon^{1/2}$

$$= g_0 \int_0^\infty \epsilon^{3/2} \bar{n}_{FD}(\epsilon)\, d\epsilon$$



$$= g_0 \left[\bar{n}_{FD}(\epsilon) \frac{\epsilon^{5/2}}{5/2} \right]_0^\infty$$



$$- \frac{2}{5} g_0 \int_0^\infty \epsilon^{5/2} \left(\frac{d\bar{n}_{FD}}{d\epsilon} \right) d\epsilon$$

$$= + \frac{2}{5} g_0 \int_0^\infty \epsilon^{5/2} \left(- \frac{d\bar{n}_{FD}}{d\epsilon} \right) d\epsilon$$

$$\bar{n}_{FD}(\epsilon) = \frac{1}{e^{+(\epsilon-\mu)/kT} + 1}$$

$$\approx \frac{2}{5} g_0 \int_{-\infty}^{+\infty} t^{5/2} \left(- \frac{d\bar{n}_{FD}}{d\epsilon} \right) d\epsilon$$

let $x \equiv \frac{\epsilon - \mu}{kT}$ $\rightarrow$ $\epsilon = (kT)x + \mu$

$$\bar{n}_{FD} = \frac{1}{e^x + 1}$$

$$\frac{d\bar{n}_{FD}}{dx} = \frac{-e^x}{(e^x+1)^2}$$

$$= \frac{2}{5} g_0 \int_{-\infty}^{+\infty} (kTx + \mu)^{5/2} \frac{e^x}{(e^x+1)^2} \, dx(kT)$$

$$= \frac{2}{5} g_0 kT \int_{-\infty}^{\infty} (kT)^{5/2} \left(x + \frac{\mu}{kT}\right)^{5/2} \underbrace{\frac{e^x}{(e^x+1)^2}}_{\text{peaks only at } x=0.} dx$$

$$\left(x + \frac{\mu}{kT}\right)^{5/2} = \left(\frac{\mu}{kT}\right)^{5/2} + \frac{5}{2}\left(x + \frac{\mu}{kT}\right)^{3/2}\Big|_{x=0} (x) + \frac{5}{2}\cdot\frac{3}{2}\left(x + \frac{\mu}{kT}\right)^{1/2}\Big|_{x=0} \frac{x^2}{2} + O(x^3)$$

$$= \frac{2}{5} g_0 (kT)^{7/2} \int_{-\infty}^{+\infty} \left(\left(\frac{\mu}{kT}\right)^{5/2} + \frac{5}{2}\left(\frac{\mu}{kT}\right)^{3/2} x + \frac{15}{4}\left(\frac{\mu}{kT}\right)^{1/2} \frac{x^2}{2} + O(x^3) \right) \frac{e^x}{(e^x+1)^2} dx$$

$$= \frac{2}{5} g_0 (kT)^{7/2} \left(\frac{\mu}{kT}\right)^{5/2} \underbrace{\int_{-\infty}^{\infty} \frac{e^x}{(e^x+1)^2} dx}_{+1} + 0$$

$$+ \frac{2}{5} g_0 (kT)^{7/2} \cdot \frac{15}{4}\left(\frac{\mu}{kT}\right)^{1/2} \cdot \frac{1}{2} \int_{-\infty}^{+\infty} x^2 \frac{e^x}{(e^x+1)^2} dx$$

$$= \frac{2}{5} g_0 \mu^{5/2} (kT) + \frac{3}{4} g_0 \mu^{1/2} (kT)^3 \cdot \frac{\pi^2}{3} + \cdots$$

$$\therefore U = \frac{2}{5} g_0 \mu^{5/2} (kT) + \frac{\pi^2}{4} g_0 \mu^{1/2} (kT)^3 + \cdots$$

w/ $g_0 = \frac{3N}{2\epsilon_F^{3/2}}$ we get

$$\Rightarrow U = \frac{Z}{5}\cdot\frac{3N}{2\epsilon_F^{3/2}}\cdot\mu^{5/2}(kT) + \frac{\pi^2}{4}\cdot\frac{3N}{2\epsilon_F^{5/2}}\cdot\mu^{1/2}(kT)^3 + \cdots$$

$$= \frac{3}{5}\frac{N}{\epsilon_F^{3/2}}\mu^{5/2}(kT) + \frac{3\pi^2}{8}\frac{N}{\epsilon_F^{3/2}}\mu^{1/2}(kT)^3 + \cdots \quad \text{eq 7.67} \checkmark$$

$$\Rightarrow U = \frac{3}{5}\frac{N}{\epsilon_F^{3/2}}\mu^{5/2}(kT) + \frac{3\pi^2}{8}\frac{N}{\epsilon_F^{3/2}}(kT)^3$$

$\left\{\frac{\mu}{\epsilon_F} = 1 - O((kT)^2)\right.$

different exponent ...

$$U = \frac{3}{5}N\epsilon_F\left(\frac{\mu}{\epsilon_F}\right)^{5/2} + \frac{3\pi^2}{8}\frac{N}{\epsilon_F}(kT)^3 + \cdots$$

Assum spin

$$= \frac{3}{5}N\epsilon_F\left(1 - \frac{\pi^2}{12}\left(\frac{kT}{\epsilon_F}\right)^2\right)^{5/2} + \frac{3\pi^2}{8}\frac{N}{\epsilon_F}(kT)^2$$

$$= \frac{3}{5}N\epsilon_F\left(1 - \frac{5\pi^2}{24}\left(\frac{kT}{\epsilon_F}\right)^2\right) + \frac{3\pi^2}{8}\frac{N}{\epsilon_F}(kT)^2$$

$$= \frac{3N}{5}\epsilon_F + \left(-\frac{\pi^2}{8} + \frac{3\pi^2}{8}\right)\frac{N}{\epsilon_F}(kT)^2 \quad \text{eq 7.68} \checkmark$$

$$\frac{\pi^2}{4}$$

Prob 7.30 All odd terms would be zero since the integrals are all taken from $(-\infty, +\infty)$

The T^4 term could be evaluated easily enough

Prob 7.31 From problem 7.28

$$g(\epsilon) = \frac{1}{2}\left(\frac{8\pi A}{h^2}\right) \quad \text{independent of } \epsilon.$$

Then

$$U = \int_0^\infty g(\epsilon)\,\epsilon\,\bar{n}_{FD}(\epsilon)\,d\epsilon = \frac{1}{2}\left(\frac{8\pi A}{h^2}\right)\int_0^\infty \epsilon\,\bar{n}_{FD}(\epsilon)\,d\epsilon$$

$$= \frac{1}{2}\left(\frac{8\pi A}{h^2}\right)\left[\epsilon\,\bar{n}_{FD}(\epsilon)\frac{\epsilon^2}{2}\Big|_0^\infty + \frac{1}{2}\int_0^\infty \epsilon^2\left(-\frac{d\bar{n}_{FD}}{d\epsilon}\right)d\epsilon\right]$$

$$= \frac{1}{4}\left(\frac{8\pi A}{h^2}\right)\int_0^\infty \epsilon^2\left(-\frac{d\bar{n}_{FD}}{d\epsilon}\right)d\epsilon \qquad \bar{n}_{FD} = \frac{1}{\left(e^{(\epsilon-\mu)/kT} + 1\right)}$$

$$= \frac{1}{4}\left(\frac{8\pi A}{h^2}\right)\int_0^\infty \epsilon^2\,\frac{e^x}{(e^x+1)^2}\,d\epsilon \qquad x \equiv \frac{(\epsilon-\mu)}{kT} \qquad \epsilon = \mu + kTx$$

$$= \frac{1}{4}\left(\frac{8\pi A}{h^2}\right)\int_{-\infty}^{\infty} \epsilon^2\,\frac{e^x}{(e^x+1)^2}\,d\epsilon = \frac{1}{4}\left(\frac{8\pi A}{h^2}\right)\int_{-\infty}^{\infty}(\mu+kTx)^2\,\frac{e^x}{(e^x+1)^2}\,(kT)\,dx$$

$$= \frac{(kT)}{4}\left(\frac{8\pi A}{h^2}\right)(kT)^2\int_{-\infty}^{\infty}\left(x+\frac{\mu}{kT}\right)^2\frac{e^x}{(e^x+1)^2}\,dx$$

$$= \left(\frac{2mA}{h^2}\right)(kT)^3 \int_{-\infty}^{+\infty} \left(\left(\frac{\mu}{kT}\right)^2 + 2\left(\frac{\mu}{kT}\right)x + 2\frac{x^2}{2}\right)\frac{e^x}{(e^x+1)^2}\,dx$$

$\pi^2/3$

$$= \left(\frac{2mA}{h^2}\right)(kT)^3 \left[\left(\frac{\mu}{kT}\right)^2 \cdot 1 + 0 + \int_{-\infty}^{+\infty} x^2 \frac{e^x}{(e^x+1)^2}\,dx \right]$$

$$= \frac{2mA}{h^2}\left[\mu^2(kT) + \frac{\pi^2}{3}(kT)^3 \right]$$

(I think this is good $\forall$ kT)

How Big is the factor of ϵ_F = ?

P. 7.32

(7.53) $$N = \int_0^\infty g(\epsilon) \frac{1}{\left(e^{(\epsilon-\mu)/kT}+1\right)}\, d\epsilon$$

(7.54) $$U = \int_0^\infty g(\epsilon)\epsilon \frac{1}{\left(e^{(\epsilon-\mu)/kT}+1\right)}\, d\epsilon$$

(a) If $kT \equiv \epsilon_F$

(7.53) $$N = \int_0^\infty g_0 \epsilon^{1/2} \frac{1}{\left(e^{(\epsilon-\mu)/\epsilon_F}+1\right)}\, d\epsilon = g_0 \int_0^\infty \epsilon^{1/2} \frac{d\epsilon}{\left(e^{\epsilon/\epsilon_F}+1\right)}$$

Question is can I exclude the integrals above zero

$$N = g_0 \int_0^\infty \epsilon^{1/2} \frac{d\epsilon}{e^{(\epsilon-\mu)/kT}+1} = g_0 \int_0^\infty \frac{\epsilon^{1/2}\, d\epsilon}{e^{(\epsilon-\mu)/kT}+1}$$

let $x = \frac{\epsilon-\mu}{kT}$ $\quad \epsilon = \mu + kTx$

$$N = g_0 \int_{-\frac{\mu}{kT}}^\infty (\mu+kTx)^{1/2} \frac{(kT)\,dx}{e^x+1} = g_0 (kT)^{3/2} \int_{-\frac{\mu}{kT}}^\infty \left(x+\frac{\mu}{kT}\right)^{1/2} \frac{dx}{e^x+1}$$

$$N(kT=\epsilon_F, \mu=0) = g_0 \epsilon_F^{3/2} \int_0^\infty x^{1/2} \frac{dx}{e^x+1}$$

(b) ... ?

P. 7.33

(a) $$g(\epsilon) = \begin{cases} g_0\sqrt{\epsilon - \epsilon_c} & \epsilon > \epsilon_c \\ g_0\sqrt{\epsilon_v - \epsilon} & \epsilon < \epsilon_v \\ 0 & \text{otherwise} \quad \epsilon_v < \epsilon < \epsilon_c \end{cases}$$

$$N = \int_0^\infty g(\epsilon)\, \bar{n}_{FD}(\epsilon)\, d\epsilon$$

$$= \int_0^{\epsilon_v} g_0\sqrt{\epsilon_v - \epsilon}\, \bar{n}_{FD}(\epsilon)\, d\epsilon + 0 + \int_{\epsilon_c}^\infty g_0\sqrt{\epsilon - \epsilon_c}\, \bar{n}_{FD}(\epsilon)\, d\epsilon$$

$$N = \int_0^{\epsilon_v} \frac{g_0\sqrt{\epsilon_v - \epsilon}\, d\epsilon}{e^{(\epsilon-\mu)/kT} + 1} + \int_{\epsilon_c}^\infty \frac{g_0\sqrt{\epsilon - \epsilon_c}}{e^{(\epsilon-\mu)/kT} + 1}\, d\epsilon$$

Don't know why μ is must be between ϵ_v & ϵ_c for a solution to this problem

Assume $\epsilon_c - \epsilon_v \gg kT$

(b) Am I supposed to actually evaluate these integrals?

(c) Si

$1\,cm^3$ of Si is approx same size as SiO_2 which is $22.69\,cm^3$

$\therefore \frac{1\,cm^3}{22.69\,cm^3} =$ ratio *

$= \#$ of Si atoms = his "4" available e^-

$=$

For Cu w/ ρ $7.12\,cm^3/mole$

$\frac{1\,cm^3}{7.12} \sim 3$ times larger than Si

But w/ How many conduction electrons? ≈ 11.

$\therefore$ Cu As about 33 times as many conduction e^-.

(d) Don't know how to get #'s

(e) I would guess $E_C - E_V \gg kT$

say $E_C - E_V = \Delta E \approx 10kT$

$kT = (8.617 \cdot 10^{-5}\ eV/K)(300K) = 24 \cdot 10^{-3}\ eV$

$= .024\ eV$

P. 7.34

(a) ?

(b)

$$N_c = \int_{\epsilon_c}^{\infty} \frac{g_{oc}\sqrt{\epsilon-\epsilon_c}}{e^{(\epsilon-\mu)/kT}+1}\,d\epsilon$$

Assum $\epsilon_c - \mu \gg kT$

$\frac{(\epsilon_c-\mu)}{kT} \gg 1$

$$N_c = g_{oc}\int_{\epsilon_c}^{\infty} \frac{\sqrt{\epsilon-\epsilon_c}\; e^{-(\epsilon-\mu)/kT}}{1+e^{-(\epsilon-\mu)/kT}}\,d\epsilon$$

$$= g_{oc}\int_{\epsilon_c}^{\infty} \sqrt{\epsilon-\epsilon_c}\; e^{-(\epsilon-\mu)/kT}\left[1 \mp e^{-(\epsilon-\mu)/kT} + \cdots\right] d\epsilon$$

$$= g_{oc}\int_{\epsilon_c}^{\infty} \sqrt{\epsilon-\epsilon_c}\; e^{-(\epsilon-\mu)/kT}\,d\epsilon$$

$$- g_{oc}\int_{\epsilon_c}^{\infty} \sqrt{\epsilon-\epsilon_c}\; e^{-2(\epsilon-\mu)/kT}\,d\epsilon + \cdots$$

Just to get a "feel" for what things look like

lets use the 1st term

$$N_c \approx g_\alpha \int_{\epsilon_c}^{\infty} \sqrt{\epsilon - \epsilon_c}\, e^{-(\epsilon-\mu)/kT}\, d\epsilon$$

let $x = \dfrac{\epsilon - \mu}{kT}$ $\quad dx = \dfrac{1}{kT} d\epsilon$

$$\epsilon = kTx + \mu$$

$$N_c \approx g_\alpha \int_{x_c}^{\infty} \sqrt{kTx + \mu - \epsilon_c}\, e^{-x}\, d\epsilon$$

$$= g_\alpha (kT)^{1/2} \int_{x_c}^{\infty} \sqrt{x + \left(\frac{\mu - \epsilon_c}{kT}\right)}\, e^{-x} (kT)\, dx$$

$$= g_\alpha (kT)^{3/2} \int_{x_c}^{\infty} \sqrt{x - x_c}\, e^{-x}\, dx$$

let $\xi = x - x_c$ Then $x = \xi + x_c$

$$N_c = g_\alpha (kT)^{3/2} \int_0^{\infty} \xi^{1/2} e^{-\xi - x_c}\, d\xi$$

$$= g_\alpha (kT)^{3/2} e^{-x_c} \underbrace{\int_0^{\infty} \xi^{1/2} e^{-\xi}\, d\xi}_{\#}$$

(c) $$N_V = \int_0^{\epsilon_V} g_{0V}\sqrt{\epsilon_V - \epsilon}\; \bar{n}_{FD}(\epsilon)\, d\epsilon$$

$$= \int_0^{\epsilon_V} g_{0V}\sqrt{\epsilon_V - \epsilon} \cdot \frac{1}{\left(e^{+(\epsilon-\mu)/kT} + 1\right)}\, d\epsilon$$

$(\mu - \epsilon_V) \gg kT \Rightarrow -\frac{(\epsilon_V - \mu)}{kT} \gg 1$

or $-(\epsilon_V - \mu) \gg kT$

$\epsilon_V - \mu \ll -kT$

$\frac{\epsilon_V - \mu}{kT} \ll -1 \Rightarrow e^{-(\epsilon_V-\mu)/kT} \gg 1 = \mathcal{O}(1)$

$e^{(\epsilon_V-\mu)/kT} \ll 1$

N_V = # of valence e^-.

$N_{holes} = N_V(T=0) - N_V(T\neq 0)$

$n_{FD}(T=0)$ = step fn at $\epsilon = \mu$.

$$N_V \approx \int_{-\mu/kT}^{\frac{\epsilon_V-\mu}{kT}} g_{0V}\sqrt{\epsilon_V - \epsilon}$$

$$x \equiv \frac{\epsilon - \mu}{kT}$$

$$N_V \approx g_{0V}\int_0^{\epsilon_V} \sqrt{\epsilon_V - \epsilon}\left[1 - e^{(\epsilon-\mu)/kT} + \cdots\right] d\epsilon$$

1st term is trivial.

2nd term

$$-g_{0V}\int_0^{\epsilon_V}\sqrt{\epsilon_V-\epsilon}\;e^{(\epsilon-\mu)/kT}\,d\epsilon$$

$$x=\frac{\epsilon-\mu}{kT} \qquad dx=\frac{d\epsilon}{kT}$$

$$\epsilon=\mu+kTx$$

$$=-g_{0V}\int_{\frac{0-\mu}{kT}}^{\frac{\epsilon_V-\mu}{kT}}\sqrt{\epsilon_V-\mu-kTx}\;e^{x}\,kT\,dx$$

$$=-g_{0V}(kT)\ \ldots$$

(d) $N_{total}=N_V+N_C(\mu)$

invert $\mu=\mu(N)$

(e) ...

P. 7. 35

(a) ? What should I use for

(b) ?

(c) ?

P. 7. 36

(a) Due to their thermal motion I would think that the spin states would be in constant flux i.e. always changing and ∴ the magnetic moments would tend to be somewhere between $+\mu_B$ & $-\mu_B$ ≈ 0.

(b) $$P_{\uparrow}(\ ) = \frac{e^{-\mu B/kT}}{\sum e^{-\mu B/kT} + \sum e^{+\mu B/kT}}$$

(c) ?

p293 Schroeder

(7.37) integral in Planck spectrum is

$$f(x) = \frac{x^3}{e^x - 1}$$

$$f'(x) = \frac{3x^2}{e^x-1} - \frac{x^3(e^x)}{(e^x-1)^2} = 0 \quad \text{set}$$

$x = 0 \qquad x \neq 0$

$$\frac{3}{(e^x-1)} - \frac{xe^x}{(e^x-1)^2} = 0$$

multiply by $(e^x-1)^2$

$$\Rightarrow \quad 3(e^x-1) - xe^x = 0$$

$$3e^x - 3 - xe^x = 0$$

$$(3-x)e^x - 3 = 0$$

(7.38) $$u(\epsilon;T) = \frac{8\pi}{(hc)^3} \frac{\epsilon^3}{(e^{\epsilon/kT} - 1)}$$

$k = 1.381 \cdot 10^{-23}$

$8.617 \cdot 10^{-5}$ eV/k

...

(7.39) $$\frac{U}{V} = \frac{8\pi}{(hc)^3}\int_0^\infty \frac{\epsilon^3 d\epsilon}{e^{\epsilon/kT}-1}$$

$f\lambda = c$

$\epsilon = h\nu = hf = \frac{hc}{\lambda}$ $\qquad d\epsilon = -\frac{hc}{\lambda^2}d\lambda$

$\lambda = \frac{hc}{\epsilon}$

$$\frac{U}{V} = \frac{8\pi}{(hc)^3}\int_\infty^0 \frac{(hc/\lambda)^3(-hc/\lambda^2)\,d\lambda}{e^{hc/\lambda kT}-1} = \frac{8\pi}{(hc)^3}\cdot(hc)^3(hc)\int_0^\infty \frac{d\lambda}{\lambda^5(e^{hc/\lambda kT}-1)}$$

$$= 8\pi(hc)\int_0^\infty \frac{d\lambda}{\lambda^5(e^{hc/\lambda kT}-1)}$$

Spectrum $U(\lambda) = \dfrac{1}{\lambda^5(e^{\frac{hc/kT}{\lambda}}-1)}$

Why does the peak not occur at λ_c

Assume $F(x)$ has max at $x_c \Rightarrow F'(x_c) = 0$

$G(y) \equiv F(x(y))$ $\qquad x(y) = \frac{1}{y}$

$G'(y) = F'(x)$

$$F(x) = F_1(x) F_2(x)$$

$$F'(x) = 0$$

$$G(y) \equiv F(x(y)) \qquad y = \frac{c}{x}$$

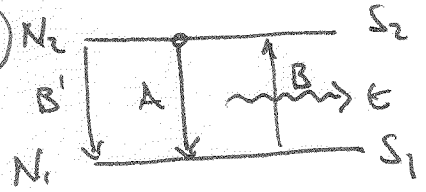
$$G'(y) = \frac{dF_1}{dx} \cdot \frac{dx}{dy} F_2 + F_1 \frac{dF_2}{dy} \cdot \frac{dx}{dy} = 0$$

set

$$\frac{dx}{dx}\left[\frac{dF_1}{dx} \cdot F_2 + F_1 \frac{dF_2}{dx} \right] = 0$$

7.40 ?

7.41



P

$$B = \frac{\text{prob of abs}}{u(f)}$$

(a) ~~$\frac{dN_1}{dt} = +AN_1 + B - BN_1 + B'N_2$~~

$$\frac{dN_2}{dt} = \frac{dN_1}{dt} = AN_1 - Bu(f)N_1 + B'u(f)N_2$$

$$\frac{N_1}{N_2} \approx C$$

~~Eq~~

~~$\frac{dN_1}{dt}$~~ & $\frac{dN_2}{dt} = -\frac{dN_1}{dt}$

How should rate of change be given?

(7.42)

(a) $U = V\left(\frac{8\pi^5}{15}\frac{(kT)^4}{(hc)^3}\right)$

(b) $u(\epsilon) = \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}$

(c) $1 = \int$

$\lambda_{min} \approx 400\,nm \rightarrow \epsilon_{max} = \frac{hc}{\lambda} = \cdots$

$\lambda_{max} \cong 700\,nm \rightarrow \epsilon_{min} = \frac{hc}{\lambda_{max}} = \cdots$

$$F = \int_{\epsilon_{min}}^{\epsilon_{max}} \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}\,d\epsilon$$

(7.43) $T = 5800\,K$

(a) $U = V\cdot\left(\frac{8\pi^5}{15}\right)\frac{(kT)^4}{(hc)^3}$

(b) $u(\epsilon) = \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}$

(c) . . .

P. 7. 44

(a) $$N = 2\sum_{n_x}\sum_{n_y}\sum_{n_z}\bar{n}_{pol}(\epsilon) = 2\sum_{n_x,n_y,n_z}\frac{1}{(e^{hcn/2LkT}-1)}$$

$$= 2\int_0^\infty dn \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\phi\, n^2 \sin\theta \frac{1}{(e^{hcn/2LkT}-1)}$$

$$= 2\left(\frac{\pi}{2}\right)\int_0^{\pi/2}\sin\theta\, d\theta \int_0^\infty \frac{n^2\,dn}{(e^{hcn/2LkT}-1)}$$

$$= \pi\left[-\cos\theta\right]_0^{\pi/2}\int_0^\infty \frac{n^2\,dn}{(e^{hcn/2LkT}-1)} \qquad \epsilon = \frac{hcn}{2L}$$

$$d\epsilon = \frac{hc}{2L}dn$$

$$= \pi\left[+1\right]\cdot\int_0^\infty \frac{\frac{4L^2}{h^2c^2}\epsilon^2\left(\frac{2L}{hc}\right)d\epsilon}{e^{\epsilon/kT}-1}$$

$$= \pi\,\frac{8L^3}{(hc)^3}\int_0^\infty \frac{\epsilon^2\,d\epsilon}{e^{\epsilon/kT}-1} \qquad \text{let } x = \frac{\epsilon}{kT}$$

$$dx = \frac{1}{kT}d\epsilon$$

$$= \frac{8\pi V}{(hc)^3}\int_0^\infty \frac{(kT)^2x^2\,(kT)\,dx}{e^x-1}$$

$$N = \frac{8\pi (kT)^3 V}{(hc)^3} \int_0^\infty \frac{x^2 dx}{e^x - 1} = \frac{8\pi (kT)^3 V}{(hc)^3} I$$

(b) $$S(T) = \frac{32\pi^5}{45} V \left(\frac{kT}{hc}\right)^3 k$$

So $$s = \frac{S}{N} = \frac{\frac{32}{45}\pi^5 V \left(\frac{kT}{hc}\right)^3 k}{8\pi \frac{(kT)^3}{(hc)^3} V \cdot I}$$

$$= \frac{4}{45}\pi^4 \left(\frac{1}{I}\right) \cdot k$$

entropy per photon is constant.

(c) $$\frac{N}{V} = 8\pi \left(\frac{kT}{hc}\right)^3 \cdot I$$

P.7.45

$$P = -\left.\frac{\partial U}{\partial V}\right|_{S,N}$$

since $N = 8\pi V\left(\frac{kT}{hc}\right)^3 \cdot I \quad \rightarrow \quad \left(\frac{kT}{hc}\right)^3 = \left(\frac{N}{8\pi V I}\right)$ from P.7.44

$$U = \frac{8\pi}{15}\frac{1}{(hc)^3} V \cdot \frac{8\pi^5}{15}\frac{(kT)^4}{(hc)^4}\cdot(hc)$$

$$= V \cdot \frac{8\pi^5}{15}(hc)\left(\frac{N}{8\pi V I}\right)^{4/3}$$

$$= V^{1-4/3} \cdot \frac{8\pi^5}{15}(hc)\frac{N^{4/3}}{(8\pi I)^{4/3}}$$

$$\left.\frac{\partial U}{\partial V}\right|_{S,N} = \left(1-\tfrac{4}{3}\right)V^{-4/3} \cdot \frac{8\pi^5}{15}(hc)\frac{N^{4/3}}{(8\pi I)^{4/3}}$$

$$= \left(1-\tfrac{4}{3}\right)\frac{V^{1-4/3}}{V}\cdot\left(\frac{8\pi^5}{15}(hc)\frac{N^{4/3}}{(8\pi I)^{4/3}}\right)$$

$$= \left(1-\tfrac{4}{3}\right)\frac{U}{V} = -\frac{1}{3}\frac{U}{V}$$

so $$P = -\left.\frac{\partial U}{\partial V}\right|_{S,N} = \frac{1}{3}\frac{U}{V}$$

P.7.46

(a) $F \equiv U - ST$

$= V \cdot \frac{8\pi^5 (kT)^4}{15(hc)^3} - \frac{32\pi^5}{45} V \left(\frac{kT}{hc}\right)^3 kT$

$= \frac{8}{15} V \pi^5 \frac{(kT)^4}{(hc)^3} \left[1 - \frac{4}{3}\right]$

$-\frac{1}{3}$

$F = -\frac{8}{45} V \pi^5 \frac{(kT)^4}{(hc)^3}$

(b) $S = -\left.\frac{\partial F}{\partial T}\right|_V = -\frac{\partial}{\partial T}\left[\cdots\right]$

$= +\frac{32}{45} V \pi^5 \left(\frac{(kT)}{(hc)}\right)^3 k.$ ✓

(c) $\left.\frac{\partial F}{\partial V}\right|_T = \frac{\partial F}{\partial V}$

$F \equiv U - ST$

$dF = dU - TdS - SdT$

$$dU = TdS - pdV$$

$$dF = TdS - pdV - TdS - SdT = -pdV - SdT$$

$$\therefore \quad p = -\left.\frac{\partial F}{\partial V}\right|_S \quad + \quad S = -\left.\frac{\partial F}{\partial T}\right|_V$$

given $S = \frac{32}{45} V \pi^5 \left(\frac{kT}{hc}\right)^3 k \quad \rightarrow \quad \left(\frac{kT}{hc}\right) = \left(\frac{45}{32}\frac{1}{\pi^5}\frac{1}{V}\cdot\frac{1}{k}\cdot S\right)^{1/3}$

$$p = -\left.\frac{\partial}{\partial V}\right|_S \left[\frac{-8}{45} V \pi^5 \frac{(kT)^4}{(hc)^3}\right]$$

$$= -\left.\frac{\partial}{\partial V}\right|_S \left[\frac{-8}{45} V \cdot \pi^5 (hc) \left(\frac{45}{32}\cdot\frac{1}{\pi^5}\frac{S}{V\cdot k}\right)^{4/3}\right]$$

$$= \left.\frac{\partial}{\partial V}\right|_S \left[\frac{8}{45} V^{1-\frac{4}{3}} \pi^5 (hc)\left(\frac{45}{32}\frac{1}{\pi^5}\frac{S}{k}\right)^{4/3}\right]$$

$$= \frac{8}{45}\frac{V^{1-\frac{4}{3}}}{V}\cdot\left(1-\frac{4}{3}\right)\ldots\ldots$$

$$= \frac{\left(1-\frac{4}{3}\right)}{V}\cdot\frac{8}{45} V \pi^5 (hc)\left(\ldots\ldots\right)^{4/3}$$

$$= \frac{1}{3 \cdot V} \cdot \frac{8}{45} V \pi^5 (hc) \left(\frac{kT}{hc}\right)^4$$

$$= \frac{1}{3 \cdot V} \quad \cdots \text{ check } \cdots$$

(d) $F = -kT \ln Z$

$$Z = \sum_i e^{-\epsilon_i / kT} \qquad \epsilon_n = \frac{hcn}{2L}$$

$$= \sum_n \exp\left\{-\frac{hcn}{2L \cdot kT}\right\} = \sum_n \left(e^{-\frac{hc}{2L \cdot kT}}\right)^n$$

$$= \frac{1}{1 - e^{-\frac{hc}{2LkT}}}$$

$$F \stackrel{?}{=} kT \ln\left(1 - e^{-\frac{hc}{2LkT}}\right) \quad \text{What's wrong?}$$

P. 7.47 [illegible] $r_{pH} = 10^9$

? How do?

P. 7.48 ν, $\bar{\nu}$ $T_{eff} = 1.95\,\mathrm{K}$

(a) $\mu_\nu = \mu_{\bar{\nu}}$ $\nu + \bar{\nu} \leftrightarrow 2\gamma$

By equating chemical potentials for a reaction of this type

$\mu_\nu + \mu_{\bar{\nu}} = \mu_\gamma = 0$ chemical potential of light is zero

→ & fact that $\mu_\nu = \mu_{\bar{\nu}}$ → [illegible] $\mu_\nu = 0$

(b) fermion $\bar{n}_{FD}(\epsilon) = \dfrac{1}{e^{\epsilon/kT} - 1}$

$\overline{N} = \sum^{3} \int \epsilon\, \bar{n}_{FD}(\epsilon)\, d\epsilon$ How set up?

(c) $N = 3 \int_\epsilon \bar{n}_{FD}(\epsilon) \cdot n(\epsilon)\, d\epsilon$

↑ prob. state w/ energy ϵ is occupied. ↑ degeneracy of state!

(d) ?

7.49 $\epsilon = \sqrt{(pc)^2 + (mc^2)^2}$ $p = [0, \infty)$

(a) $U = 2^2 \int_0^\infty \epsilon \, n_{F.D}(\epsilon) \, d\epsilon$ $n_{FD}(\epsilon) = \dfrac{1}{e^{\epsilon/kT} + 1}$

$= 4 \int_0^\infty \dfrac{\epsilon \, d\epsilon}{e^{\epsilon/kT} + 1}$ $\epsilon = \sqrt{(pc)^2 + (mc^2)^2}$

$\epsilon^2 = (pc)^2 + (mc^2)^2$

$= 4 \int_0^\infty \cdots$

(b) $u(T) = \int_0^\infty \dfrac{x^2 \sqrt{x^2 + (mc^2/kT)^2}}{e^{\sqrt{x^2 + (mc^2/kT)^2}} + 1} \, dx$

(d) $F = -kT \ln Z$

P. 7.50 $\frac{kT}{c^2} \gg m$

(a) $V(t)$ T $F = U - ST$

$\Rightarrow S = \frac{F-U}{T}$

$$S = -\frac{16\pi (kT)^4}{(hc)^3} f(T) \cdot V - \frac{16\pi (kT)^4}{(hc)^3} U(T) \cdot V$$

(b) ?

(c) ?

(7.51) $T = 3000\ K$

$e = 1/3$

(a) $P = 100\ W.$

using the $\text{power} = \sigma e A T^4$

$$100\ W = \left(5.67\cdot10^{-8}\ \frac{W}{m^2 K^4}\right)\left(\frac{1}{3}\right)(A)(3000\ K)^4$$

$$100\ W = \frac{3^4\cdot10^{12}}{3}(A)\ 5.67\cdot10^{-8}\ \frac{W}{m^2}$$

$$100\ W = 3^3\cdot10^{12}\cdot A\cdot 5.67\cdot10^{-8}\ W/m^2$$

$$A = \frac{100}{3^3\cdot10^{12}(5.67\cdot10^{-8})}\ m^2 = 6.532\cdot10^{-5}\ m^2$$

$$= .6532\cdot(\mu m)^2$$

$(\mu m)^2 = 10^{-6}\ m$

(b) $\epsilon = 2.82\,kT$

$= 2.82(1.381\cdot10^{-23}\ J/K)(300\ K)$

$= 1.1683\cdot10^{-20}\ J = 0.0729\ eV$

$\epsilon = h\nu = hf = \frac{hc}{\lambda}$ or $\lambda = \frac{hc}{\epsilon}$

$$\lambda = \frac{(6.626\cdot10^{-34}\ J\cdot s)(3\cdot10^{8}\ m/s)}{1.1683\cdot10^{-20}\ J}$$

$$= 1.701\cdot10^{-5}\ m$$

(c) Spectrum of light $u(\epsilon) = \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}$ energy density per unit photon energy

$u(\epsilon)$ = Now

$$\frac{8\pi}{(hc)^3} = \frac{8\cdot\pi}{[(6.626\cdot10^{-34}\ J\cdot s)(3\cdot10^{8}\ m/s)]^3} = 3.1998\cdot10^{75}$$

$$= 3.1998\cdot10^{75}\ \frac{\epsilon^3}{}$$

$$kT = (1.381\cdot10^{-23}\ J/k)(3000\ k) = 4.143\cdot10^{-20}\ J$$

$$\therefore\ u(\epsilon) = (3.1998\cdot10^{75})\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}$$

let $x \equiv \epsilon/kT$

$$u(x) = \frac{(3.1998\cdot10^{75})(kT)^3 x^3}{(e^x-1)}$$

$$u(x) = (2.2755 \cdot 10^{17}) \frac{x^3}{(e^x - 1)}$$

– – –

Spectrum of light $\quad u(\epsilon) = \frac{8\pi}{(hc)^3} \frac{\epsilon^3}{(e^{\epsilon/kT} - 1)}$

$$\epsilon = h\nu = \frac{hc}{\lambda}$$

$$u(\lambda) = \frac{8\pi}{(hc)^3} \frac{(hc)^3 (\lambda^{-3})}{e^{\frac{hc}{\lambda kT}} - 1} = \frac{8\pi}{\lambda^3} \frac{1}{\left(e^{\left(\frac{hc}{kT}\right)\cdot\frac{1}{\lambda}} - 1\right)}$$

$$\frac{hc}{kT}\Big|_{T=3000k} = \frac{(6.626 \cdot 10^{-34} J \cdot s)(3 \cdot 10^8 m/s)}{(1.381 \cdot 10^{-23} J/k)(3000 k)} = 4.798 \cdot 10^{-6} m$$

$$= 4.798 \text{ um} \equiv \lambda_0$$

$$u(\lambda) = \frac{8\pi h}{\lambda^3} \frac{1}{(e^{\lambda_0/\lambda} - 1)}$$ plot in units of λ_0

$\lambda \sim 0 \quad \frac{\infty}{\infty} \cdot \frac{1}{\infty} = \frac{\infty}{\infty}$ indeterminant guess $\rightarrow 0$.

$$I(T) = \int_{400\,nm}^{700\,nm} U(\lambda)\,d\lambda = \ldots$$ Calculated radiant energy per unit time? in the visible spectrum.

(e)

(f) Find Maximum of $I_{vis}(T)$ intensity of radiated energy per unit time.

P. 7. 52

(a) $P = \sigma e A T^4$ $\quad T \approx 300\,k$

$A_{Body} \cong 1\,m^2$

$e = 1$

$\sigma = 5.67 \cdot 10^{-8} \frac{W}{m^2 k^4}$

$\Rightarrow P = \left(5.67 \cdot 10^{-8} \frac{W}{m^2 k^4}\right)(1)(1\,m^2)(300\,k)^4 = 459.27\,W$

What is surface area of Body?

(b) $E/_{day} = P \cdot (3600)(24) = 3.968092 \cdot 10^7\,J$ radiated in photons.

$Cal/_{day} = 2000\ kilocal = 8.3736 \cdot 10^6\,J$ ingested as calories.

to look for humans get T_H exactly + then max return from a Black Body at T_H is $\epsilon \approx 2.8 k T_H = \frac{hc}{\lambda}$ $\Rightarrow$ tells one how to build

$f = \frac{c}{\lambda}$ a sensor to look for people.

Don't know?

(c) $\left(\frac{P}{M}\right)_{Sun} = \frac{3.9 \cdot 10^{26}\,W}{(2 \cdot 10^{30}\,kg)} = 1.95 \cdot 10^{-4}\,W/kg$

$\left(\frac{P}{M}\right)_{Human} = \frac{459.27\,W}{81.64\,kg} = 5.62\,W/kg$

7.53

(a) $T_{BH} =$

$$\epsilon_{peak} = 2.8kT_{BH} = 2.8(1.381\cdot10^{-23}\,J/k)(\quad\cdot h$$

$$\epsilon_{peak} = 2.8k\frac{hc^3}{16\pi^2 kGM} = \frac{2.8\,hc^3}{16\pi^2 GM}$$

$$= \frac{2.8(6.626\cdot10^{-34}\,J\cdot s)(3\cdot10^8\,m/s)^3}{16\pi^2(6.673\cdot10^{-11}\,N\cdot m^2/kg^2)(2\cdot10^{30}\,kg)}$$

$$= 2.3769\cdot10^{-30}\,\frac{J\cdot s}{N\cdot m^2}\,\frac{m^3}{s^2}\,kg$$

$$= \text{"}\quad \frac{J\cdot kg}{N\cdot s^2} = J. \qquad J = F\cdot L = N\cdot m \qquad N = kg\cdot\frac{m}{s^2}$$

$\epsilon = \epsilon$ $\qquad \epsilon = h\nu = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{\epsilon}$$

$$\lambda_{peak} = \frac{hc}{\frac{2.8\,hc^3}{16\pi^2 GM}} = \frac{16\pi^2}{2.8}\frac{G\cdot M}{c^2} = 8.86\cdot10^4\,m$$

$$SA = 4\pi R^2 \qquad R = \left(\frac{SA}{4\pi}\right)^{1/2} = 4\pi^{1/2}\frac{GM}{c^2}$$

$$= 1.05\cdot10^4\,m$$

(b) $P = \sigma e A T^4$

$$= \sigma e \left(\frac{16\pi G^2 M^2}{c^4} \right) \left(\frac{\hbar c^3}{16\pi^2 k G M} \right)$$

$$= \frac{\sigma e}{\pi c} \frac{G M}{k} = 5.8 \cdot 10^{26} \text{ W}$$

(c) $E = Mc^2$

$$\frac{dE}{dt} = c^2 \frac{dM}{dt} = P = \left(\frac{\sigma e}{\pi c} \frac{G}{k} \right) \cdot M$$

$$\frac{dM}{dt} = \left(\frac{\sigma e}{\pi c^3} \frac{G}{k} \right) M$$

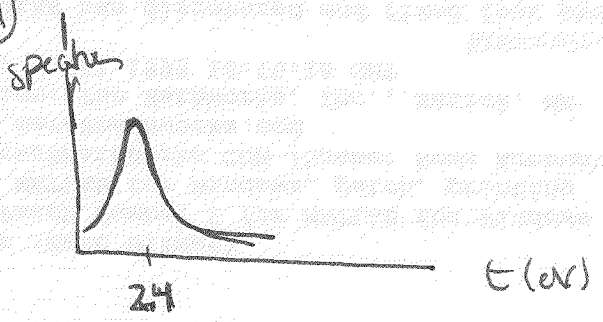
$$M = M_0 \exp\left\{ \beta t / \frac{\pi c^3 k}{\sigma e G} \right\}$$

$$t_0 \equiv \frac{\pi c^3 k}{\sigma e G} = 3.09 \cdot 10^{20} \text{ s} = 9.7 \cdot 10^{12} \text{ years}$$

(e) $M_0 = ?$ M ...

P.7.54

(a)



$$E_{peak} = 2.8kT \quad \Rightarrow \quad T = \frac{2.4\ eV}{2.8(8.617\cdot10^{-5}\ eV/K)} = 9.94\cdot10^{3}\ K$$

$$P = \sigma e A T^4 \qquad P = \frac{(2.4\cdot3.9\cdot10^{26}\ W)}{(5.67\cdot10^{-8}\ \frac{W}{m^2K^4})(1)(A)}$$

$$2.4\cdot3.9\cdot10^{26}\ W = (5.67\cdot10^{-8}\ \tfrac{W}{m^2K^4})(1)A\,(9.94\cdot10^{3}\ K)^4$$

$$A = 1.69\cdot10^{19}$$

$4\pi R^2 = A$

$$R = 1.16\cdot10^{9}\ m = 1.16\cdot10^{6}\ km$$

$R_{sun} = 7\cdot10^{8}\ m$ close 10 x larger

(b) $P = (.03)(3.9\cdot10^{26}\ W)$

$$E_{peak} = 2.8kT = 7\ eV$$

$$\Rightarrow T = \frac{7\ eV}{2.8(8.617\cdot10^{-5}\ eV/K)} = 29000\ K$$

$$P = \sigma e A T^4$$

$$\Rightarrow A = \frac{P}{\sigma e T^4} = \frac{(.03)(3.9\cdot10^{26}\ W)}{(5.67\cdot10^{-8}\ \frac{W}{m^2K^4})(1)(29\cdot10^{3}\ K)^4} =$$

04-17-03 ?

$A = 2.9 \cdot 10^{14}\ m^2$

$\Rightarrow R = 4 \cdot 10^6\ m$. About $\frac{1}{100}$ radius of our sun

(c) $\epsilon = 2.8kT = 0.8eV$

$T = \frac{0.8\ eV}{(2.8)(8.617 \cdot 10^{-5}\ eV/K)} = 3300K.$

$\epsilon = 0.8eV = h\nu = \frac{hc}{\lambda}$

$\Rightarrow \lambda = \frac{hc}{\epsilon} = \frac{(4.136 \cdot 10^{-15}\ eV \cdot s)(3 \cdot 10^8\ m/s)}{.8\ eV} = 1.5\ \mu m$

visible light $\sim$ 400 nm – 700m $\Rightarrow$ .4 μm – .7 μm

$P = (10^4 \cdot 3.9 \cdot 10^{26}\ W) = 5.67 \cdot 10^{-8} \cdot e \cdot A \cdot T^4$

$\Rightarrow A = \frac{P}{5.67 \cdot 10^{-8} \cdot e \cdot T^4} = 5.8 \cdot 10^{23}\ m^2$

$\Rightarrow R = 2.1 \cdot 10^{11}\ m$ $\qquad R_{sun} = 7 \cdot 10^8$

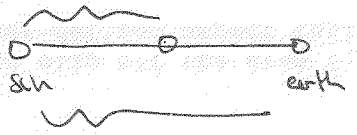
$\approx$ 1000 times greater than our sun.

Prob 7.57 ?

P. 7.86 Infrared light responsible for

$R = .7(150 \cdot 10^6 \text{ km}) = 105 \cdot 10^6 \text{ km}$

(a)



$R = 150 \cdot 10^6$ km

$P = 3.9 \cdot 10^{26}$ W

Solar constant from Venus $= \dfrac{P}{4\pi(R^2)} = \dfrac{3.9 \cdot 10^{26} \text{ W}}{4\pi (105 \cdot 10^6 \cdot 10^3)^2 \text{ m}^2}$

$= 2.8 \cdot 10^3 \text{ W/m}^2$

(a) $\Rightarrow$ (1)

Amt heat absorbed by

$(\text{solar constant})\pi R^2 = \varepsilon\sigma \cdot \dfrac{A}{\pi R^2} \cdot T^4 = \varepsilon\sigma 4\pi R^2 \cdot T^4$

$\Rightarrow T = \left(\dfrac{2800 \text{ W/m}^2}{4\pi (5.67 \cdot 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4})} \right)^{1/4} =$ [illegible] K.

333.3 K

(b) Assuming on 30% of incident flux gets through

$$T = \left(\frac{(.3)(2800\ W/m^2)}{\sigma} \right)^{1/4} = \cdots \quad 246.69$$

(c) ?

P. 8.57 eq 7.112 $U =$

eq 7.108 is

$$U = 3\int_0^{n_{max}} dn \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\phi\, n^2 \sin\theta \frac{\epsilon}{e^{\epsilon/kT} - 1}$$

$$= [\text{illegible}]$$

$$= 3\left(\frac{\pi}{2}\right)(-\cos\theta)\Big|_0^{\pi/2} = \frac{3\pi}{2}(1) = \frac{3\pi}{2}$$

$$= \frac{3\pi}{2}\int_0^{n_{max}} dn\, n^2 \left(\frac{hcn}{2L}\right)\frac{1}{(e^{hcn/2LkT} - 1)}$$

let $x = \frac{hcn}{2LkT}$ $\rightarrow$ $n = \frac{2LkT}{hc}\cdot x$

$$dn = \frac{2LkT}{hc}\,dx$$

$$= \frac{3\pi}{2}\int_0^{x_{max}} \left(\frac{2LkT}{hc}\right)\cdot dx \left(\frac{2LkT}{hc}\right)^2 x^2 \,[\text{illegible}]\,(xkT)\,\frac{1}{(e^x - 1)}$$

$$= \frac{3\pi}{2}\left(\frac{2LkT}{hc}\right)^3 \int_0^{x_{max}} \frac{x^3\,dx}{e^x - 1}\cdot kT$$

$$= \frac{3\pi}{2}\left(\frac{2LkT}{hc_s}\right)^3 kT \int_0^{x_{max}} \frac{x^3\,dx}{e^x-1}$$

$$x_{max} = \frac{hc_s}{2LkT}\cdot\left(\frac{6N}{\pi}\right)^{1/3} = \frac{hc_s}{2kT}\left(\frac{6N}{\pi V}\right)^{1/3} \equiv \frac{T_D}{T}$$

Then $$T_D = \frac{hc_s}{2k}\left(\frac{6N}{\pi V}\right)^{1/3} \qquad \left(\frac{2k}{hc_s}\right) = \frac{1}{T_D}\left(\frac{6N}{\pi V}\right)^{1/3}$$

$U =$ [illegible]

$$= \frac{3\pi}{2}\cdot L^3T^3 \,[\text{illegible}]\left(\frac{2k}{hc_s}\right)^3 kT \int_0^{x_{max}} \frac{x^3\,dx}{e^x-1}$$

$$= \frac{3^2\pi}{2}\,\frac{1}{T_D^3}\left(\frac{6N}{\pi V}\right) L^3T^3\,kT \int_0^{x_{max}} \frac{x^3\,dx}{e^x-1}$$

$$= [\text{illegible}]\;3^2\left(\frac{T}{T_D}\right)^3 NkT \int_0^{T_D/T} \frac{x^3\,dx}{e^x-1}$$

$$= \frac{9NkT^4}{T_D^3}\int_0^{T_D/T} \frac{x^3\,dx}{e^x-1}$$ eq 7.112 ✓

$$C_V = \frac{\partial U}{\partial T}$$

$$U = \frac{3\pi}{2} \int_0^{n_{max}} \frac{hc_s}{2L} \frac{n^3}{\left(e^{hc_s n/2LkT} - 1\right)} dn$$

$$\frac{\partial U}{\partial T} = \frac{3\pi}{2} \int_0^{n_{max}} \frac{hc_s}{2L} \frac{n^3}{\left(e^{hc_s n/2LkT} - 1\right)^2} e^{\frac{hc_s n}{2LkT}} \left(\frac{hc_s n}{2LkT^2}\right)(-1)\, dn$$

$$\frac{\partial U}{\partial T} = -\frac{3\pi}{2} \frac{hc_s}{2L} \left(\frac{hc_s}{2LkT^2}\right) \int_0^{n_{max}} \frac{n^4 e^{\frac{hc_s n}{2LkT}}}{\left(e^{hc_s n/2LkT} - 1\right)^2} dn$$

let $x = \frac{hc_s n}{2LkT}$ $\quad n = \frac{2LkT}{hc_s} x$ $\quad dn = \left(\frac{2LkT}{hc_s}\right) dx$

$$\frac{\partial U}{\partial T} = -\frac{3\pi}{2} \left(\frac{hc_s}{2L}\right)^2 \frac{1}{kT^2} \int_0^{x_{max}}$$

...

P. 7.58 $c_s = 3560\ m/s$

$$T_D = \frac{hc_s}{2k}\left(\frac{6N}{\pi V}\right)^{1/3} \qquad V = 7.12\ cm^3$$

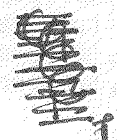
$$N = N_A = 6.02 \cdot 10^{23}$$

$$= \frac{(6.626\cdot10^{-34}\ J\cdot s)(3560\ m/s)}{2(1.381\cdot10^{-23}\ J/k)}\left(\frac{6.602\cdot10^{23}}{\pi(7.12\cdot10^{-6}\ m^3)}\right)^{1/3}$$

$$= 465.0686\ k.$$

Experimentally:

How do I measure T_D from this plot?



Slope of curve $= \frac{12\pi^4 Nk}{5T_D^3}$

$$\Rightarrow \quad = \frac{1.8 - .9}{16 - 2} = \frac{.9}{14} = .0643\ \frac{mJ}{k^4}$$

$$= .0643\cdot10^{-3}\ \frac{J}{k^4}$$

So that

$$\left(\frac{1}{T_D}\right)^3 = \frac{5}{12\pi^4 N_A k}\left(.0643\cdot10^{-3}\ J/k^4\right)$$

$$T_D = 311.5\ k$$

P. 7.59 According to the free electron model of specific heat $C_V = \ldots$

$$\gamma = \frac{\pi^2 N k^2}{2\epsilon_F}$$

If ϵ_F is approximately the same $\forall$ species then the y-intercept will be 0.

P.7.60 $$C_V = \gamma T + \frac{12\pi^4 N k}{5 T_D^3} T^3 \qquad T \ll T_D$$

$C_V = \gamma T$ $\qquad \gamma = \frac{\pi^2 N k^2}{2\epsilon_F}$ $\qquad T_D = 465.06\ k$

$$\epsilon_F = \frac{\hbar^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3} = \ldots$$

$$\gamma T = \frac{12\pi^4 N k}{5 T_D^3} T^3$$

$$T^2 = (\quad)\gamma$$

P.7.61

P. 7.61

$^4He = 4.0 + 2 \text{ neutrons}$

$\approx 6.0 \text{ g/mole} = M$ atomic weight

$$C_V = Nk\left(\frac{T}{4.67\,K}\right)^3 \qquad T < 0.6\,K$$

$c_s = 238 \text{ m/s}$ $\qquad \rho = 0.145 \text{ g/cm}^3$ $\qquad V = \frac{M}{\rho}$

$\frac{N}{V} = \frac{nN_A}{V} =$

$V =$ (1) liter / mole of ^4He

(2) $N = N_A$

(3) $V = \frac{m}{\rho} = \frac{[M]}{[\rho]} = \frac{g}{(g/cm^3)}$

$\therefore \frac{N}{V} = \frac{N_A \rho}{M}$

n moles $= nM$ grams $= \frac{N}{N_A} M$

$C_V = ?$ $\qquad C_V = \frac{12\pi^4}{5}\left(\frac{T}{T_D}\right)^3 Nk$

$$T_D = \frac{\hbar c_s}{2k_B}\left(\frac{6N}{V}\right)^{1/3}$$

(1) compute $T_D = \cdots$

(2) compute C_V

P. 2.62 7.112

$$U = \frac{9NkT^4}{T_D^3}\int_0^{T_D/T}\frac{x^3\,dx}{e^x-1}$$

$$\frac{x^3}{e^x-1} = x^3\cdot\frac{1}{(e^x-1)}$$

Let $f(x) = \dfrac{x^3}{e^x-1}$

$$f(0) = \left.\frac{3x^2}{e^x}\right|_{x=0} = 0 \qquad \frac{"0"}{"0"}$$

$$f'(x) = \frac{3x^2}{e^x-1} + \frac{x^3(-1)e^x}{(e^x-1)^2} = \frac{x^2}{(e^x-1)^2}\left[3(e^x-1)+x(-1)e^x\right]$$

$$= \frac{x^2}{(e^x-1)^2}(3e^x-3-xe^x) = \frac{x^2}{(e^x-1)^2}\cdots$$

P. 7.63 ?

P. 7.64 $T \propto \left(\frac{1}{\lambda}\right)^2$

$E = hf$ + $p = \frac{h}{\lambda}$ for particles

$E \propto p^2$ for magnons

$E = \frac{p^2}{2m^*}$ $m^* = \text{constant} = 1.24 \cdot 10^{-29}\ kg$

$= 14 \cdot m_e$

(a) ?

(b) ?

↓

P. 7.64

$\epsilon = hf \qquad p = \frac{h}{\lambda}$

$\epsilon^2 = \frac{p^2}{2m^*} = \frac{h^2}{2m^*}\frac{1}{\lambda^2} \qquad \lambda = \frac{2L}{n}$ same as for photons

(a) $N = g\sum_n \bar{n}_n(\epsilon) = \int_0^\infty n \ \ldots$ Not entirely sure

(b) $\frac{M(0) - M(T)}{M(0)} = \frac{2\chi_B N - 2\chi_B N_m}{2\chi_B N}$

$= 1 - \frac{N_m}{N} = \ldots$

(c)

P. 7.65 eq 7.124

$$\int_0^\infty \frac{\sqrt{x}\,dx}{e^x - 1} = \cdots$$

P. 7.66

(a) $\epsilon_0 =$

(b) $kT_c = (0.527)\left(\frac{h^2}{2\pi m}\right)\left(\frac{N}{V}\right)^{2/3}$

?

(d) ?

P. 7.67 ?

P. 7.68 ?

P. 7.69

(a) $N = \int_0^\infty g(\epsilon)\,\frac{d\epsilon}{e^{\epsilon - \mu}/kT - 1}$ let $t = T/T_c$ + $x = \frac{\epsilon}{kT_c}$

$c = \frac{\mu}{kT_c}$

$$= \int_0^\infty g(xkT_c)\,\frac{(dx/kT_c)}{e^{(xkT_c - ctT_c)/ktT_c} - 1} \qquad d\epsilon = \frac{dx}{kT_c}$$

$$= \int_0^\infty \frac{g(xkT_c)}{kT_c}\,\frac{dx}{e^{(x-c)/t} - 1}$$

04-23-01 ?

7.122 is

P. 7.70

(a) ?

P. 7.71 ?

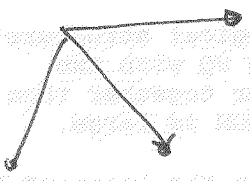
P. 7.72 ?

P. 7.73 (a) ?

P. 7.74 ?

P. 7.75 ?

$$Z_{ideal} = \frac{1}{N!}\left(\frac{V Z_{int}}{V_Q}\right)^N \qquad V_Q = \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3$$



$\binom{n}{2} = \frac{n(n-1)}{2}$ connections for a network of n nodes

$$Z_c = \frac{1}{V^N}\int d^3r_1 \cdots d^3r_N \prod_{pairs} e^{-\beta u_{ij}}$$

$$e^{-\beta u_{ij}} = 1 + f_{ij}$$

$$\prod_{pairs} e^{-\beta u_{ij}} = \prod_{pairs}(1+f_{ij}) = (1+f_{12})(1+f_{13})(1+f_{14})\cdots(1+f_{1N})(1+f_{23})(1+f_{24})\cdots$$

$$= 1 + \sum_{\substack{pairs \\ (connections)}} f_{ij} + \sum_{\substack{distinct \\ pairs}} f_{ij}f_{kl} + \cdots$$

$$\frac{1}{V^N}\int d^3r_1 \cdots d^3r_N \, T_{12} = \frac{1}{V^N} V^{N-2} \int d^3r_1 d^3r_2 \, T_{12}$$

$$= \frac{1}{V^2}$$

$$\frac{1}{V^2}\int d^3r_1 \, d^3r_2 \, T_{12} \quad \frac{N(N-1)}{2}$$

$$\frac{N(N-1)(N-2)}{3!}$$

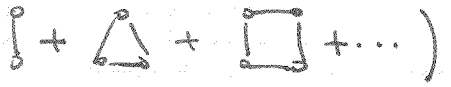
~~$\frac{N(N-1)}{2}$~~ ~~$N(N-1)$~~

$$\frac{N(N-1)(N-2)(N-3)}{8}$$

$$Z = Z_{ideal} \cdot Z_c$$

$$F = -kT \ln Z = -kT \ln Z_{ideal} - kT \ln Z_c$$

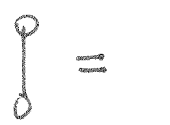
$$= -NkT \ln\left(\frac{V}{NV_Q}\right) - kT\left(\; + \; + \; + \cdots \right)$$



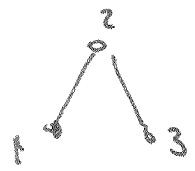
$$P = -\left.\frac{\partial E}{\partial V}\right|_{N,T} =$$

$$F = U - TS$$

P.8.1



$$= \frac{N(N-1)}{2V^2} \int d^3r_1 d^3r_2 f_{12}$$



$$= \frac{N(N-1)(N-2)}{2V^3} \int d^3r_1 d^3r_2 d^3r_3 f_{12} f_{23}$$

Don't follow the symmetry factor very well

$$= \frac{N(N-1)(N-2)(N-3)}{2V^4} \int d^3r_1 d^3r_2 d^3r_3 d^3r_4 f_{12} f_{34}$$

$$= \frac{N(N-1)(N-2)}{4 \cdot 6 V^3} \int d^3r_1 d^3r_2 d^3r_3 f_{12} f_{23} f_{31}$$

~~4~~ $2 \cdot 2 = 4$

P.8.2 ...

P.8.3 ...

P.8.4 ...